Sixty-Six Theses: Next Steps and the Way Forward in the Modified Cosmological Model

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Abstract

The purpose is to review and lay out a plan for future inquiry pertaining to the modified cosmological model (MCM) and its overarching research program. The material is modularized as a catalog of open questions that seem likely to support productive research work. The main focus is quantum theory but the material spans a breadth of physics and mathematics. Cosmology is heavily weighted and some Millennium Prize problems are included. A comprehensive introduction contains a survey of falsifiable MCM predictions and associated experimental results. Listed problems include original ideas deserving further study as well as investigations of others' work when it may be germane. A longstanding and important conceptual hurdle in the approach to MCM quantum gravity is resolved. A new elliptic curve application is presented. With several exceptions, the presentation is high-level and qualitative. Formal analyses are mostly relegated to the future work which is the topic of this book. Sufficient technical context is given that third parties might independently undertake the suggested work units.

Contents

0 Introduction

This book contains a long list of thesis problems in physics and mathematics. A previous review [1] was written to broaden the horizons of the modified cosmological model (MCM) and the present purpose is to pinpoint within those horizons ideas that should be brought forward to completion.

0.1 Review and Main Results

During the MCM's main development phase, this writer had already exited the academic environment which is most conducive to initial surveys of topics concluding in original contributions at the level of a PhD thesis. The fixation of this research program on the bare fundamentals has come at the expense of such "PhD level" work. This condition provides fodder for detractors. Thus, remediation is in order.

While the fractional distance program in real analysis [2] must exceed the requirements for a PhD in mathematics, this writer has rarely taken research in physics to a conclusive calculation, and never at the level of a PhD thesis. In the way that mathematicians are sometimes said to be concerned with the existence of solutions more so than with finding them, it follows that this writer's thesis equivalent [2] is in real mathematical analysis. The presumed existence of solutions has sufficed throughout the MCM's development, contrary to what is most common in physics. First and foremost, however, this writer is a physicist. Physics ultimately requires real solutions for experimental applications. It was hoped for many years that others would jump at the chance to write the papers in which such solutions are given but history has taken a different tack. In light of events, the present work describes many open and untreated questions that have arisen in the development of the MCM.

An early computation in the MCM found a characteristic length scale for new physics at 10^{-4}m [3]. As it is the aim of this research program to tie up physics' loose ends with a new model of cosmology (and ontology with quantum applications), the characteristic scale was obtained when the structure of the MCM was applied in an intuitive way to the foremost unsolved problem in classical mechanics: the precession of spinning discs. If any theory will be a *theory of everything*, it will lay to rest the open questions in classical mechanics. Thus, an MCM mechanism for anomalous mechanical precession was supposed. The calculation yielding 10^{-4}m was very simple but, on the other hand, precession is not a manifestly complicated problem. The result was remarkable because 10^{-4}m is neither the nano-scale of quantum mechanics nor the macro-scale of classical mechanics. Instead, an intermediate meso-scale was obtained in the regime where catch-all *losses due to friction* are usually called on to scoop up everything neither classically deterministic nor quantum mechanical. Furthermore, Arkani-Hamed and others have already written about the open question of new physics at the sub-millimeter scale [4, 5]. Is it only a coincidence that 10^{-4}m lies in the narrow strip where new physics is not forbidden? This question deserves further study because the result cannot be ruled out immediately.

The mechanism surrounding the scale calculation in [3] was well-defined but possibly not as well motivated as is expected in professional publications. One reason for this is that this writer is not a professional. As an unpaid contributor, he is not constrained by the professional community standards which sometimes make it difficult to put highly speculative ideas to paper. Still, the 10^{-4}m result is remarkable. If 10^{-30}, 10^{-10}, 10^{0}, or 10^{10}m was determined as the scale for the mechanism proposed in [3], then we could know without any further thinking that the mechanism was unphysical. To the contrary, the computation shows that experiment allows the idea, in part, at least. If the work of physicists is to rule out theories, which are only ideas or *formalized* ideas, this calculation shows that the MCM passes at least one hurdle of its falsifiable predictions not being ruled out. The hurdle was not high but first hurdles rarely are.

The best prediction to come from the MCM is that there should not exist any spin-0 fundamental particles such as the Higgs boson. This prediction is directly falsifiable in a way that exceeds the possibility for new effects on a certain scale. The prediction is perfectly well motivated [6]. It is as clean and concise as any prediction in the history of physics. It arose in the following line of inquiry. After a brief review of Kaluza–Klein theory (KKT), the MCM unit cell was constructed in [7]. The purpose of the construction was to build on previous work in the MCM so as to address some of the failures of KKT detailed by Overduin and Wesson [8]. Namely, the so-called *cylinder condition* requires that 4D Kaluza–Klein physics in spacetime must not depend on the fifth coordinate. This condition is generated or satisfied in the MCM when the realm of physics is taken as a 4D Poincaré section (slice) of a 5D space for some constant value of the fifth coordinate. This is the condition set by the MCM unit cell (Section 0.2). Another problem is that KKT only allows solutions in which the electromagnetic (EM) strength tensor $F_{\mu\nu}$ vanishes. While one 5D Kaluza–Klein (KK) metric tensor contains an EM potential 4-vector and a dual 4-vector, the MCM uses two such metrics containing twice as many degrees of freedom. This doubling of the degrees of freedom should be sufficient for $F_{\mu\nu} \neq 0$ solutions.

While much work remains to formalize the MCM at the level of Kaluza's and Klein's original papers [9, 10], the MCM unit cell was assembled in [7] to address KKT's main problems. Soon after, it was demonstrated that the unit cell offers a good answer to the fundamental question of quantum field theory (QFT) [6]. That question asks why we have the particles we have and not some other particles. The *standard model* of particle physics is pretty good for determining what our particles do but it does nothing to address the fundamental question about why we have the standard model particles to begin with. In the MCM, the spectrum of lattice vibrations in the unit cell is identical to the known spectrum of elementary particles. Thus, the spectrum of fundamental particles results from a fundamental geometric structure underlying reality. Even such nuance as the eight varieties of gluons arises in the MCM lattice from simple classical mechanics. Each particle is given as a different kind of spring or mass in a 5D lattice of masses connected by springs. The ultimate goal of QFT is to generate the true spectrum of fundamental particles from theory itself without having to force agreement with experiment by the imposition of an empirical model, i.e.: the standard model. The MCM's main disagreement with the standard model is in the scheme for fundamental bosons. The standard model supposes that there exists a spin-0 fundamental particle: the famous scalar boson following from the work of famous people such as Englert, Brout, Higgs, Guralnik, Hagen, and Kibble [11–17]. The MCM scheme does not, in its current incarnation, permit the existence of any spin-0 fundamental particles. So, the MCM answer to the fundamental question of QFT is plainly falsifiable.

Posed in early 2013, the prediction that all fundamental bosons should have spin-1 followed on the heels of the discovery of a new particle at CERN in 2012 [18, 19]: the Higgslike particle. If that particle is found have spin-0, then the MCM is wrong and it needs to be revised or scrapped. If that particle is the Higgs boson, or if it is any possible variety of Higgs boson, it will have spin-0. The objective existence of a spin-0 fundamental particle would send an important MCM result back to the drawing board. More than causing a rescission of a prediction, the entire structure of the model would be cast into doubt. As it stands, the MCM is supposed to generate the fundamental particles as lattice vibrations in an almost (but not quite) trivial model of lattice cosmology. The truthfulness of this prediction requires that the Higgslike particle has spin-1.

Though many detractors of the MCM cite an alleged mountainous body of evidence proving that the Higgslike particle does not and cannot have spin-1, Ralston has shown that spin-1 was not ruled out by the initial observations at CERN [20]. Arkani-Hamed has also stated in a talk [21] that spin-1 is not ruled out for the Higgslike particle. Ralston, in his analysis of the decay channels reported by CERN, cites "model-independent Lorentz invariance" as allowing spin-1. In the ten years since the particle was discovered, this writer has not seen a treatment of the model independent amplitudes cited by Ralston. Instead, the ATLAS collaboration rules out "some specific models" of spin-1 [22], "several alternative spin scenarios" [23], and "alternative hypotheses for spin" [24]. The CMS collaboration reports that, "all tested spin-one boson hypotheses are excluded," [25] and, "any mixed-parity spin-one state is excluded" [26]. Neither collaboration reports that they have ruled out spin-1 in the model-independent case of Lorentz invariance, or even that they have studied it.

In further contradiction to the claims of certain detractors of the MCM, Particle Data Group (PDG)—the de facto bottom-line authority on the state of the art in particle physics—reports that the spin of the Higgslike particle was not yet determined as of 2020. PDG writes the following [27].

> "Whereas the observed signal is labeled as a spin-0 particle and is called a Higgs Boson, the detailed properties of H^0 and its role in the context of electroweak symmetry breaking need to be further clarified. [*sic*] The observation of the signal in the $\gamma\gamma$ final state rules out the possibility that the discovered particle has spin 1, as a consequence of the Landau–Yang theorem. This argument relies on the assumptions that the decaying particle is an on-shell resonance and that the decay products are indeed two photons rather than two pairs of boosted photons, which each could in principle be misidentified as a single photon."

Regarding the Landau–Yang theorem, experiment trumps theory. Indeed, experiments are carried out mainly with the intention to falsify theories. Landau–Yang would go out the window if an experimental result was found to disagree with it. While this theorem is well trusted, theory can never rule out reality. Ralston writes the following regarding the dominion of experiment over theory [20].

> "The Landau–Yang theorems are inadequate to eliminate spin-1. Theoretical prejudice to close the gaps is unreliable, and a fair consideration based on experiment is needed. A spin-1 field can produce the resonance

> structure observed in invariant mass distributions, and also produce the same angular distribution of photons and ZZ decays as spin-0. However spin-0 cannot produce the variety of distributions made by spin-1. The Higgs-like pattern of decay also cannot rule out spin-1 without more analysis. Upcoming data will add information, which should be analyzed giving spin-1 full and unbiased consideration that has not appeared before."

It is unusual that ten years have gone by since the particle was discovered and the "unbiased consideration" has not yet appeared in the literature (to the knowledge of this writer.) Considerations published by ATLAS [22–24] and CMS [25, 26] are biased under the suppositions of one model or another. While it seems impossible, the literature appears to suggest that the model-independent case has not yet been considered. What does seem possible is that the model-independent case *has been considered* and the result has been withheld due to politics. Indeed, we suggest that the particle is "labeled" as a spin-0 particle and "called" a Higgs boson [27] mainly to further a false impression that the MCM prediction for spin-1 has been ruled out. Usually physicists are zealously and notoriously reluctant to jump to conclusions, but not in this case.

Just months after the MCM prediction for spin-1 [6], Ellis and You wrote the following [28].

> "There are many indirect and direct experimental indications that the new particle H discovered by the ATLAS and CMS Collaborations has spin zero and (mostly) positive parity, and that its couplings to other particles are correlated with their masses. Beyond any reasonable doubt, it is a Higgs boson[.]"

This excerpt may contain the only reference in the entirety of the physics literature to the formal standard of proof in USA jurisprudence: reasonable doubt. A more common standard in physics is given by the motto of the Royal Society: *Nullius in verba*. It means "take nobody's word for it." Ellis and You make their bold and patently unscientific claim in the abstract of their paper but they back off from the outrageous overstatement in the paper's first sentence [28].

> "It has now been established with a high degree of confidence that the new particle H with mass $\sim$ 126 GeV discovered by the ATLAS and CMS [*Collaborations*] has spin zero."

This paper of Ellis and You is remarkable not only for its reference to some ill-defined and unquantifiable standard of "reasonable doubt" in place of physics' usual

5σ criterion, but also because it was the first citation of the Royal Swedish Academy of Sciences in their technical write-up regarding the 2013 Nobel Prize in Physics [29]. The prominent citation by the Royal Swedish Academy of Sciences can be construed as an endorsement of the false claim that the Higgslike particle is the Higgs boson *beyond a reasonable doubt*. Aside from the reasonable doubt cast by the MCM prediction for spin-1, Ralston has reported that an entirely indeterminate amount of doubt remains [20]. PDG cites an uncertain number of photons *and* a questionable assumption about the on-shell condition as reasonable sources of doubt. Most importantly, PDG only cites known unknowns as sources of doubt when unknown unknowns may give reason to doubt as well.

Almost two years after Ellis and You published, CMS reported with atypical bluntness that it was still important to study the spin-1 case experimentally because the observed state *may be that one* [26].

> "Despite the fact that the experimental observation of the $H \to \gamma\gamma$ decay channel prevents the observed boson from being a spin-one particle, it is still important to experimentally study the spin-one models in the decay to massive vector bosons in case that the observed state is a different one."

It is not clear whether CMS suggests (*i*) the existence of a second, different particle at ∼125GeV, (*ii*) that the observed one is different than the one ruled out by the Landau–Yang theorem, or (*iii*) that the final state is different than $\gamma\gamma$. CMS' obtuse language about "a different one" is consistent with a theme of sidestepping the spin-1 issue in the literature. Even while CMS emphasizes the importance of experimental study, they still call the $H \to \gamma\gamma$ decay a fact while PDG reports that this channel is not yet established as a fact [27]. Assuming that it is a fact, as it may be, CMS does not state their reliance on the assumed perfection of the Landau–Yang theorem to find that such a decay prevents spin-1.

If reasonable doubt were to have some meaning in physics, then it could only be the usual standard of 5σ. However, there does not exist any literature claiming to have ruled out spin-1 at that level. Certain models of spin-1 have been ruled out to certain levels, but the model-independent, objective property of spin-1 has never been ruled out for the Higgslike particle at any high significance, and never at 5σ. Most likely, the reference to the reasonable doubt standard of USA jurisprudence was used to establish in a court of USA law, for some (nefarious) reason, that this writer's prediction was wrong. In fact, spin-1 has not been ruled out. Any publication claiming that spin-1 has been ruled out will be found to have ruled out only certain models of spin-1 divorced from the case of model-independent Lorentz invariance [20].

Ten years later, one would think that the particle's discoverers would have determined its spin. To this writer's knowledge, no other particle's spin was so elusive that it could not be determined even ten years after the initial discovery. In the opinion of this writer, the Higgslike particle *has* been determined to have spin-1 and CERN withholds the result because it supports the MCM over work which is better loved in the academic mainstream.

Moving along, another falsifiable MCM prediction was posed in [30]. It was suggested that one might observe variations in the value of the fine structure constant correlated with the delay between an event and its detection in some apparatus. The unstated but implicit reasoning was that the state space of things which existed in the past is not the same as the state space of things which exist in the present. Therefore, observables might depend on how far in the past an event occurred prior to its detection. Such was already the case for an earlier MCM result regarding dark energy [31]. Distant cosmological objects appear to accelerate due to their displacement far back on the light cone (Section 7). Though the unit cell was not constructed until about a year after the quantum delay prediction appeared in [30], the unit cell elucidates the motivation for delay correlations and complements it with further motivation. Signals from events in the past are usually thought to propagate into detectors along a path in topological Minkowski space. In the MCM, in addition to an altered state space in the past [30], the past is not totally Minkowski in the unit cell. Due to the MCM's fifth dimension, one may speak of earlier *chronological times*, which are Minkowski, as well as earlier *chirological times* in which the past is topologically anti-de Sitter. (Chronological time is the timelike coordinate x^0 in 4D spacetime and chirological time is a new fifth coordinate $\chi^4_\pm$.) Propagation through some non-Minkowski geometry will cause deviations from the predictions for pure Minkowski propagation and these deviations should be correlated with the amount of time spent in the non-Minkowski geometry. This prediction is not so precise as the prediction that the Higgslike particle should have spin-1 but it is a strong prediction. If such delay correlations are not observed, then the fundamental ideation behind the prediction would be falsified. The predicted correlations *were* observed by the BaBar collaboration [32], however!

The main gist communicated here to the reader is that all of the verifiable ideation in the MCM has survived: the specific things and the less specific things. More than 99% of new theories can be rejected immediately due to some obvious physical problem so it is a great accomplishment of the MCM not to be one of those theories. Often laypersons hear that new theories are a dime a dozen, which is true, but this glosses over a further notion that is more relevant in the present case. A new theory that can survive even a cursory check is a diamond in the rough. Almost none of them make it past a single hurdle. Ones that do are often absurdly convoluted. Quintessence and the chameleon field are examples of convoluted theories being not so convoluted that they are immediately discarded. Even the modern theory of cosmological inflation, which is not easy to rule out, is rather convoluted. To the contrary, the MCM is elegant, intuitive, and simple, though not yet mathematically formalized with new equations of motion. Still, there is no trivial way to rule out the MCM, as is the case for almost all new theories. This testifies to the good quality of the work. Beyond the lack of an easy rejection, the MCM's predictions have *multiple experimental confirmations* such as the prediction for delay correlations. These confirmations obliterate detractors' persistent claims of wrongness *and* not-even-wrongness.

The BaBar experiment concluded in 2008. The primary analysis of the data generated by the experiment had also concluded by the time of the MCM delay prediction. However, the search for these correlations in the BaBar data was undertaken immediately following the publication of the MCM prediction. Not astonishingly, the MCM prediction was borne out when BaBar published their observation of time reversal symmetry violation in the B^0 meson system [32]. While the BaBar analysis did not exactly search for the delay correlations in the value of the fine structure constant α which had been suggested, the result follows. Since physics is Hamiltonian, meaning that everything is calculable once any two things are determined, the value of α which can be extracted from the delay correlations published in [32] will depend on the delay. The observation of time reversal symmetry violation is easily the 21st century's second biggest discovery in particle physics after the Higgslike particle. This discovery is a direct experimental verification of the structure of the MCM.

During the primary data analysis stage following BaBar's data collection stage, no one had the idea to check for correlations with delay. After it was suggested that the MCM would be such that delay correlations should exist, someone at BaBar checked and found a signal that had escaped detection. No one had any reason to expect such correlations but then time reversal symmetry violation was discovered and the history of physics was changed forever. If the Higgslike particle is eventually reported to have spin-1, then the 21st century's biggest and second biggest particle physics discoveries will be among the MCM's small handful of falsifiable predictions. Not only that, the MCM also predicts (among even more things) the dark energy effect whose discoverers were awarded the 2011 Nobel Prize in Physics: Perlmutter, Schmidt, and Riess [33–35]. So, there is a decent volume of ordinary physics output recorded in the publications constituting the MCM. The lack of an easy falsification among these predictions makes the MCM better than 90% of similar attempts to bushwhack a new path. The confirmation by BaBar makes the MCM the best new theory on the market today, bar none. Unfortunately, BaBar does not credit the ideation for delay correlations to this writer and the ordinary scientific proceedings are retarded.

The predictions above, and others mentioned below, are intermingled with other content in MCM publications. Some of that content is non-standard. Why the weird tone? After this writer became convinced that his work was blacklisted against appearing even on the unreviewed arXiv, a tone was adopted which could never pass peer review, even in the absence of blacklisting. Despite the presence of outstanding original work, the tone in many MCM publications is such that they could never appear in physics' usual venue for the dissemination of scientific information. Although the MCM's many grand successes form an independent rebuke, the non-standard content and tone was added as a second rebuke so that this writer could be seen doubly rebuking the establishment which prefers the political mechanisms of the USA to the actual practice of science. Following these earlier MCM publications, the present work lays out a series of problems whose write-ups should be sufficiently technical that the tone of the papers cannot be confused or conflated with the results. As mentioned above, the technical treatment of the problems should rise in many cases to the level of a PhD thesis. To date, it has been easier for detractors to conflate the author's prose with his main results due to an absence of such clearly demonstrated, PhD-level technical mastery or a commensurately voluminous set of calculations.

This writer has not been able to publish even on arXiv: the unreviewed (yet censored) preprint repository in which low quality work is published every day (along with many fair or outstanding research papers in physics and mathematics.) Before the non-standard tone was adopted, [31] was submitted to arXiv in September 2009. The typesetting and graphics were substandard, the tone was ordinary, and the content was top-tier. For some reason most likely related to a payment routed through Cyprus to Paul Manafort in October of 2009 [36], the paper was rejected for publication on arXiv.[1] Details relating to the publication status of [31] may be found in Appendix C.

The overall lack of peer review for the MCM, which is a subset of the censorship problem at arXiv and elsewhere, provides more fodder for detractors. Even the most outlandish and easily disproven models of alternative physics have extensive

[1]For placement of [31] on the spectrum of what is acceptable in the physics preprint literature, compare to [37,38], particularly Figure 12 in the latter.

online documentations including Wikipedia articles and various forum discussions, e.g.: *timecube*. The relative invisibility of the MCM on the internet suggests that the publication blacklist exceeds blacklisting in the traditional publication venues and goes so far as the total prohibition of this writer's intention to communicate results. As a scientist, a physicist's trade is to ply the scientific method whose final step is *communicate results*. The fake internet bubble in which this writer appears to communicate results while ultimately failing to do so, for the most part, has had a stronger negative impact on this writer's career than any number of stylistic writing choices ever could. Still, this writer's research does get communicated, somehow. [31] is now called SCP-001 in certain corners of the internet where the MCM is known to exist.

The supposition, or *allegation*, that the MCM has not passed peer review is false. Before moving on to a review of the MCM unit cell and its labeling conventions (Section 0.2), followed by a review of the MCM scheme for fundamental particles (Section 0.3), we will summarize the extensive peer review of the MCM and its glowing yet uncredited receptions. The MCM began as a work in phenomenology. Given certain results, a model of cosmology was constructed to accommodate them [31]. The optical effect described as dark energy was explained without an anomalous (and borderline unphysical) acceleration of the expansion of the 3D spatial universe. Instead, accelerating expansion in the time sector of 4D spacetime was identified as the cause of the observed optical effect. This was the kernel of the idea that things in the past should not be exactly as they are in the present. In [39], inquiry into the structure of the past was taken all the way back to the cosmological beginning. Since a famous theorem of Arnowitt, Deser, and Misner [40, 41] proves that the 0-component of the universe's 4-momentum must be non-zero, the usual model of big bang cosmology cannot conserve 4-momentum. Given a presumed $p^\mu = (0,0,0,0)$ before the big bang, $p^\mu(t) = (p^0, p^1, p^2, p^3)$ at $t > 0$ cannot conserve momentum if $p^0 \neq 0$. However, physics requires that momentum is conserved. In the way that Pauli was able to deduce the existence of the neutrino from a quantity of missing momentum in nuclear β decay, it was deduced that a big bang would have to spawn two universes moving oppositely through time if it was a momentum-conserving process. If the energy of one universe is positive-definite, then the other universe (whose time has a minus sign on it) would be negative-definite. This is required to conserve 4-momentum, as is usual in physics.

After the proposition for negative time was published in November 2011 [39] (as a restatement of the same idea published in 2009 [31]), Rubino et al. reported an experiment regarding negative frequency in quantum optics [42]. Since frequency is inverse time, and since the experiment was reported only months after negative time was found to resolve the momentum problem in big bang cosmology, we suggest that the ideation for the experiment of Rubino et al. followed after a review of early work in the MCM. Short of experimental verification, it is the highest and most valid form of peer review that one man's research should influence another man's research direction. Many papers passing ordinary, administrative peer review go on to accumulate zero citations but papers well received by the community of experts in that area go on to acquire citations. If not for the apparent USA-sponsored blacklist of this writer, it is suggested that Rubino et al. might have cited [31, 39] as motivating their search for physical negative frequency modes. What peer review can be higher than to have one's work received and built upon? The answer cannot be a layer of dust atop an unknown but peer reviewed CV item.

Spawning new scientific inquiry among one's peer community is nearly the highest form of peer review. It far surpasses the administrative peer review which is widely hated by academics [43] and yet revered as holy by those who are only indirectly aware of the mechanism. Surpassing even positive reception in one's community, the highest mark in peer review is experimental confirmation. Rubino et al. write the following about their discovery of negative frequency resonant radiation (NRR) [42].

> "[*F*]requency conversion processes may be understood in terms of energy transfer between specific modes [*sic*]. However, to date only the positive frequency branch of the dispersion has been considered when this actually also has a branch at negative frequencies. This branch is usually neglected or even considered meaningless when, in reality, it may host mode conversion to a new frequency. The fact that a mode on the negative branch of the dispersion relation may be excited has a number of important implications, beyond the simple curiosity of the effect in itself. Indeed, light always oscillates with both positive and negative frequencies, but the negative-frequency part is directly related to its positive counterpart and seems redundant. On the other hand, light particles, photons, have positive energies and are associated with positive frequencies only. A process such as that highlighted here, that mixes positive and negative frequencies will therefore change the number of photons, leading to amplification or even particle creation from the quantum vacuum.
>
> "In this work we show how alongside the usual resonant radiation spectral peak observed in many experiments, a second, further blue-shifted peak is also predicted. This new peak may be explained as the result of the excita-

> tion of radiation that lies on the negative frequency branch of the dispersion relation. We first explain why this radiation should be observed and then provide experimental evidence of what we call 'negative frequency resonant radiation' in both bulk media and photonic crystal fibres."

NRR is a direct confirmation of the theory of negative time at the heart of the MCM. Although the existence of these negative frequency modes had been known for a long time, no one thought to look for them until the theory of negative time was published [31, 39]. Perhaps history will show that this was only a coincidence. In any case, we suggest that the negative frequency experiment was motivated by a review of the MCM and that the experiment confirmed the negative time hypothesis through the observation of negative frequency optical modes.

To the extent that Rubino et al. cite the possibility for "amplification," consider the following from a follow-on publication of Rubino et al. in late 2012.

> "[*W*]e may derive a photon number balance equation by generalizing [*sic*] to the case of a moving scatterer. We find that:
>
> $$|\mathrm{RR}|^2 - |\mathrm{NRR}|^2 = 1,$$
>
> where $|\mathrm{RR}|^2$ and $|\mathrm{NRR}|^2$ are the photon numbers of the [*resonant radiation*] and [*negative resonant radiation*] modes normalized to the input photon number [*sic*]. The negative sign in front of the $|\mathrm{NRR}|^2$ photon number is a direct consequence of the fact that the NRR-mode has negative frequency in the comoving reference frame [*sic*]. So the difference between the normalized number of photons has to be equal to the photon number in the input mode. As a consequence, the total output photon number, $|\mathrm{RR}|^2 + |\mathrm{NRR}|^2 > 1$, i.e. ***we have amplification*** [*emphasis added*]. The scattering process mediated by the traveling [*relativistic inhomogeneity*] will amplify photons as a result of the coupling between the positive and negative frequency modes."

As we have previously commented on the eccentric citation of Ellis and You to the legal standard of doubt in USA jurisprudence, the note at the top of [44] (not excerpted) is also eccentric. It is the only instance of such a note that this writer has come across.[1] The note directs that correspondence and requests for materials should be addressed to coauthor Faccio. The eccentric note in anticipation of correspondence is given because [44] reports that the authors discovered free energy. The negative

[1] This writer does not regularly browse the experimental quantum optics literature.

frequency optical mode which follows from the negative time mode in the MCM—following logically and chronologically—revealed the holy grail of physics: a feasible mechanism for the construction of a device whose coefficient of efficiency exceeds unity. While the MCM did not predict the application in quantum optics, it follows because negative frequency is inverse negative time. It is suggested that this writer's peers saw that it follows, did the experiment, and confirmed the physics. So, the MCM has yet again passed the true bar of review by peers without passing the false bar of administrative peer review under the docents of a politicized bureaucracy. The MCM has been experimentally confirmed at least twice. If the Higgslike particle has spin-1, it will be at least three times. Next to experimental confirmation, administrative peer review is meaningless. If it was suggested that objects on Earth tend to fall in the downward direction, no one would ask if the claim has passed peer review. For the MCM, however, the fact that it has not passed peer review in the most artificial and useless sense is cited as problematic to the extent that it overrides the experimental verification.

Following the work of Rubino et al. on NRR [42,44], Lockheed abruptly announced in 2014 near-term plans for truck-sized nuclear fusion reactors [45]. Fundamentally, Lockheed was front-running their expectation for the mass production of NRR power generators which would be truck-sized because they are only optical tables in a box (in the opinion of this writer.) After Lockheed's initial press releases, the West Texas oil contract cratered in 2014 and it had not recovered as of 2021.[1] The blacklist on the MCM was extended by the powers that be to cover up the only hope by which humanity might escape its shackles of toil: a new energy source. These results regarding free energy are now known in certain corners of the internet as "golf rumors." The quoted name follows from men at their country clubs talking about the NRR result before the full violence of the USA political machine squashed such talk.

The discovery of negative frequency resonant radiation by Rubino et al. [42, 44] suggests that the MCM has passed peer review with flying colors. The result about time reversal symmetry violation published by BaBar does the same [32]. Both of these results connect to the MCM's requirement for negative time, through negative frequency and time reversal respectively. Both results are experimental confirmation of the MCM in excess of an affirmative peer review by positive reception leading to follow-on work. Additionally, there are no results which rule out the MCM predictions for new effects at 10^{-4}m, the prediction that the Higgslike particle should have spin-1, or any other features of the model. The many MCM mechanisms described in [31] are

[1] During the preparation of this manuscript, the WTI oil contract reached highs not seen since 2014.

each likely to be parlayed into further experimental confirmations. Additionally, there are many mathematical confirmations. For example, the MCM search for quantum gravity shows that Einstein's equation for general relativity may be derived in a certain quantum formalism (Section 1.10). A number alike to the fine structure constant to within 0.4% is characteristic of this formalism as well (Section 1.9). The Riemann hypothesis was falsified as a corollary of mathematical results developed for describing physics in the unit cell [2,46–48]. Other examples of affirmative review by peers include the following.

- Ashtekar's response papers [49,50] which are detailed at length in Appendix C.
- Wilczek's 2012 quantum time crystals [51,52] follow from the 2011 $\hat{M}^3$ operator developed in [30].[1] The MCM unit cell is the unit cell of a time crystal in the most intuitive way (Section 57).
- Almost all of Finkelstein's arXiv publications are MCM response papers (Section 33).
- Mochizuki's "Hodge theater" is the MCM dressed in a thick coat of jargon (Section 31).
- Hairer's $3M Breakthrough Prize-winning "regularity structure" [53] is the MCM unit cell dressed in another coat of jargon (Section 32). When Hairer's colleague reported that Hairer's work must have been done by *aliens* [54], it was a jibe regarding how obviously Hairer had used the MCM and its $\hat{M}^3$ operator without citation. Apparently, those on the far side of the MCM blacklist see something akin to aliens between them and this writer.
- The RBM model in the autodidactic universe of Alexander et al. [55] is plainly the process given by $\hat{M}^3$.

The list of such glowing yet uncited peer reviews goes on and on. It must exceed those few papers which have come to this writer's attention.

0.2 The MCM Unit Cell

This section contains a glossary of symbols pertaining to the MCM unit cell, Figure 1. Remarks on its most prominent features are given in context. Further remarks will follow. We will begin with the unit cell's metric and coordinate conventions.

[1] This writer became aware of viXra in the summer of 2012. The viXra submission dates of References [30,31,39] do not reflect the initial publication dates.

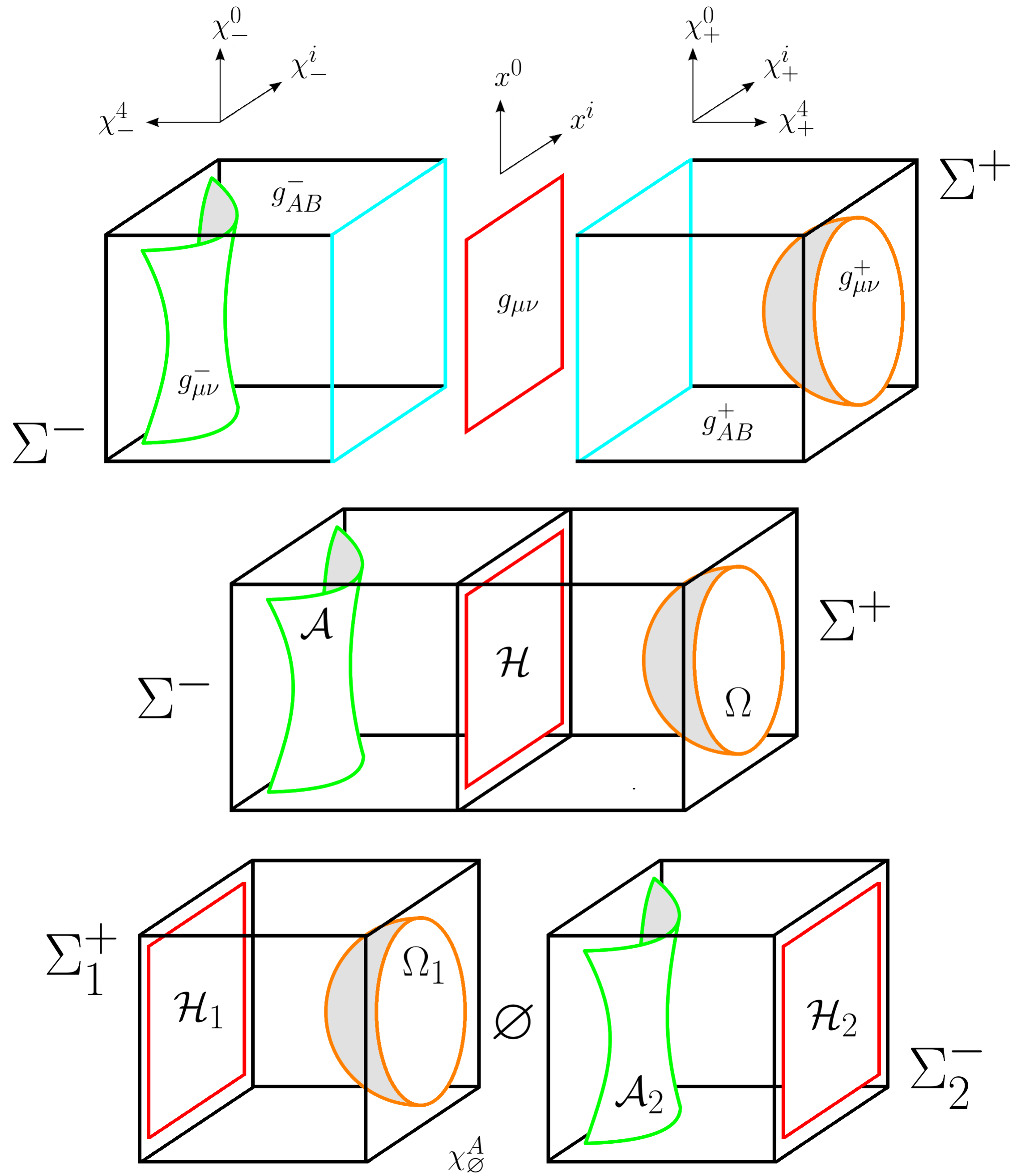

Figure 1: The MCM unit cell is the fundamental element of a cosmological lattice. $\mathcal{H}$ is a Minkowski space representing the observable universe. $\Sigma^\pm$ do not include their shared boundary at $\mathcal{H}$. It is expected that the χ_-^A coordinates are left-handed if the χ_+^A coordinates are right-handed. The second figure with $\Sigma^\pm$ joined on $\mathcal{H}$ is most properly the unit cell in the sense of crystallography but often *unit cell* will refer to the representation centered on $\varnothing$.

Notation is such that Greek tensor indices run from 0 to 3. Upper case Latin indices run from 0 to 4. Lower case Latin indices run from 1 to 3.

- A^μ is the electromagnetic potential 4-vector. This object has its usual meaning and we will usually assume $A^\mu = 0$. This facilitates consideration of the simplest cases which can be extended to $A^\mu \neq 0$ later.
- $A^\mu_\pm$ are electromagnetic potential 4-vectors in $\Sigma^\pm$. Usually, descriptions of the MCM assume an $A^\mu_\pm = 0$ ground state.
- $\Sigma^\pm$ are 5-spaces bounded in the fifth direction. The fifth coordinate is positive-definite in Σ^+ and negative-definite in Σ^-. The metric signature of $\Sigma^\pm$ is $\{-+++\pm\}$.
- $\chi^A_\pm$ are the 5D coordinates in $\Sigma^\pm$. Coordinates written with χ are called **abstract coordinates** to distinguish them from **physical coordinates** written with x. Different coordinate charts' distances are measured with different metrics. Although $\chi^4_\pm = 0$ will be undefined, the origins of $\chi^4_\pm$ are located in $\mathcal{H}$ in the sense that $\chi^4_\pm$ measure distance relative to $\mathcal{H}$. $\chi^4_\pm$ is respectively positive- or negative-definite in $\Sigma^\pm$.
- $\chi^\alpha_\pm$ are the abstract coordinates of $\Sigma^\pm$ at some constant value of $\chi^4_\pm$.
- $\chi^A_\varnothing$ or $\chi^\alpha_\varnothing$ are the hypothetical coordinates to the right of Ω and to the left of $\mathcal{A}$, as in the lower representation of Figure 1. In previous usage, $\chi^4_\varnothing$ has referred to a single point added to splice χ^4_+ with χ^4_- between Ω_1 and $\mathcal{A}_2$. Similarly, a hypothetical $\chi^4_\pm = 0$ would splice $\chi^4_\pm$ at $\mathcal{H}$. However, $\chi^4_\pm = 0$ is not defined due to the positive- and negative-definiteness of $\chi^4_\pm \in \Sigma^\pm$. The exact details for connecting Ω_1 to $\mathcal{A}_2$ form one of the major outstanding problems in the MCM. Since the **level of aleph** (Section 1.6) changes at $\varnothing$, meaning that $\varnothing$ marks the progression from one neighborhood of fractional distance to the next (Section 1.6), the pointlike property of $\chi^4_\varnothing$ on one level of aleph may be resolved in greater detail as an interval on another level of aleph. For this reason, it is supposed that $\varnothing$ might span a 5-space requiring $\chi^A_\varnothing$ coordinates rather than $\chi^\mu_\varnothing$. The exact details of $\varnothing$ are not yet fully determined.
- x^μ are the physical, relativistic coordinates of the geometric manifold $\mathcal{H}$, a Minkowski space. Distance between the points specified with x^μ is given by the metric $g_{\mu\nu}$. These coordinates have their usual meaning.

- $x^\mu_\pm$ are the physical coordinates of gravitational manifolds located in $\Sigma^\pm$ at constant values of $\chi^4_\pm$. x^μ_+ charts Ω at $\chi^4_+ = \Phi$ and x^μ_- charts $\mathcal{A}$ at $\chi^4_- = -\varphi$. Φ is the golden ratio and φ is its inverse. The Ω and $\mathcal{A}$ manifolds are also charted in the abstract $\chi^\mu_\pm$ coordinates so it is required to carefully distinguish between the physical coordinates $x^\mu_\pm$ and the abstract coordinates $\chi^\mu_\pm$. Occasionally, we may speak of $x^\mu_\pm$ as the physical coordinates at arbitrary constant values of $\chi^4_\pm$.

- $g_{\mu\nu}$ is the metric of 4D Minkowski space $\mathcal{M}_4$. If $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ with $\eta_{\mu\nu}$ the flat Lorentzian metric and $h_{\mu\nu}$ a small perturbation, we will almost always assume $h_{\mu\nu} = 0$. In the general case, this metric is to be determined from a matching condition on the metrics in $\Sigma^\pm$ where a mismatch will result in $h_{\mu\nu} \neq 0$.

- $g^\pm_{AB}$ is the 5D metric of the abstract $\chi^A_\pm$ coordinates in $\Sigma^\pm$. It is based on the Kaluza–Klein metric

$$g^{\text{KK}}_{AB} = \begin{pmatrix} g_{\alpha\beta} + \kappa^2\phi^2 A_\alpha A_\beta & \kappa\phi^2 A_\alpha \\ \kappa\phi^2 A_\beta & \phi^2 \end{pmatrix} \quad ,$$

where ϕ is a scalar field, κ is a constant, and A^μ is an EM potential 4-vector. The $g^\pm_{AB}$ metrics are obtained by identifying $\phi^2_\pm$ in $\Sigma^\pm$ with a function of the fifth abstract coordinate $\chi^4_\pm$. Setting $\kappa = 1$, we have

$$g^\pm_{AB} = \begin{pmatrix} g^\pm_{\alpha\beta} + f_\pm(\chi^4_\pm) A^\pm_\alpha A^\pm_\beta & f_\pm(\chi^4_\pm) A^\pm_\alpha \\ f_\pm(\chi^4_\pm) A^\pm_\beta & f_\pm(\chi^4_\pm) \end{pmatrix} \quad .$$

In general, we will assume that f is the identity function setting $\phi^2_\pm = \chi^4_\pm$. Taking the simplest case of $A^\mu_\pm = 0$, we have

$$g^\pm_{AB} = \begin{pmatrix} g^\pm_{\alpha\beta} & 0 \\ 0 & \chi^4_\pm \end{pmatrix} \quad .$$

In Section 7, we will show that this metric supports an MCM solution to dark energy. Since $\chi^4_\pm$ is positive or negative in $\Sigma^\pm$ respectively, $g^\pm_{AB}$ has Lorentzian signature $\{\mp \pm \pm \pm \pm\}$ in Σ^+ and pseudo-Lorentzian signature $\{\mp \pm \pm \pm \mp\}$ in Σ^-. This signature is also supported by $\phi_\pm = \chi^4_\pm$ if χ^4_- is imaginary relative to χ^4_+. Since the exact role for the MCM scalar field has not been fully developed, it will suffice to let $g^\pm_{44}$ be oppositely signed as $\chi^4_\pm$ with an understanding that we may later choose $\phi^2_\pm = \pm|\chi^4_\pm|^2$.

- $g^{\pm}_{\mu\nu}(\chi^4_{\pm})$ is the physical metric on a submanifold of $\Sigma^{\pm}$ defined by some constant value of $\chi^4_{\pm}$. This metric describes distances in the physical $x^{\mu}_{\pm}$ coordinates. When $A^{\mu}_{\pm}=0$, $g^{+}_{\mu\nu}(\chi^4_{+})$ is the dS_4 de Sitter metric in Σ^{+} and $g^{-}_{\mu\nu}(\chi^4_{-})$ is the AdS_4 anti-de Sitter metric in Σ^{-}. The dS or AdS space at a given value of $\chi^4_{\pm}$ is the one whose constant Ricci scalar R is equal to that value of $\chi^4_{\pm}$. In other words, the KK scalar field is such that ϕ^2 becomes the Ricci scalar of the maximally symmetric physical metrics. dS and AdS are called *maximally symmetric* because the Ricci scalar is constant in the manifold and the geometry is completely determined by its value. $g^{\pm}_{\mu\nu}$ will implicitly refer to the $g^{+}_{\mu\nu}(\Phi)$ physical metric on Ω and the $g^{-}_{\mu\nu}(-\varphi)$ physical metric on $\mathcal{A}$.

To explain how the metric $g_{\mu\nu}$ in $\mathcal{H}$ should be obtained from the $g^{\pm}_{AB}$ metrics, we will make reference to a scale factor which has not been introduced yet. It will be covered in Section 1. We want $g_{\mu\nu}$ to be a superposition of contributions from $g^{\pm}_{AB}$, as in [7]. It should be the superposition of the limits of the 5D metrics as $\chi^4_{\pm}\to 0$. Letting $A^{\mu}_{\pm}=0$ and assigning scale factors Φ and φ to $g^{\pm}_{AB}$, the scaled sum of $g^{\pm}_{AB}$ is

$$\Phi g^{+}_{AB}+\varphi g^{-}_{AB}=\begin{pmatrix}\Phi g^{+}_{\alpha\beta}+\varphi g^{-}_{\alpha\beta} & 0\\ 0 & \Phi\chi^4_{+}+\varphi\chi^4_{-}\end{pmatrix} \quad .$$

In the $\chi^4_{\pm}\to 0$ limit, the fifth diagonal position vanishes. The fifth position is associated with the Ricci scalar and $R=0$ defines Minkowski space. While the fifth diagonal position may have additional physics associated with its context as a scalar field, the metric in $\mathcal{H}$ is presently defined as the 4D part of the metric superposition:

$$g_{\mu\nu}=\Phi g^{+}_{\alpha\beta}+\varphi g^{-}_{\alpha\beta}=\begin{pmatrix}-\Phi c^2 & 0 & 0 & 0\\ 0 & \Phi & 0 & 0\\ 0 & 0 & \Phi & 0\\ 0 & 0 & 0 & \Phi\end{pmatrix}+\begin{pmatrix}-\varphi c^2 & 0 & 0 & 0\\ 0 & \varphi & 0 & 0\\ 0 & 0 & \varphi & 0\\ 0 & 0 & 0 & \varphi\end{pmatrix} \quad .[1]$$

To obtain a natural scale for the metric in $\mathcal{H}$, we might rephrase the expression as a difference but instead we will appeal to the sign freedom in the $\{\mp\pm\pm\pm\}$ metric

[1] Part of the reason for leaving $\chi^4_{\pm}=0$ undefined is to avoid a picture of $g_{\mu\nu}$ as the 4D part of a metric whose fifth diagonal position vanishes.

signature. We give the opposite sign convention to g^-_{AB} to obtain

$$g_{\mu\nu} = \Phi g^+_{\alpha\beta} + \varphi g^-_{\alpha\beta} = \begin{pmatrix} -\Phi c^2 + \varphi c^2 & 0 & 0 & 0 \\ 0 & \Phi - \varphi & 0 & 0 \\ 0 & 0 & \Phi - \varphi & 0 \\ 0 & 0 & 0 & \Phi - \varphi \end{pmatrix} = \eta_{\mu\nu} \quad ,$$

where $\eta_{\mu\nu}$ is the perturbation free case of $g_{\mu\nu}$ in signature $\{-+++\}$. Due to the opposite sign conventions for the metrics in $\Sigma^\pm$, the metric in $\mathcal{H}$ is both a superposition and a solitonic difference like a shadow cast by $g^\pm_{AB}$. This metric structure is expected to become rich when one adds non-zero $A^\mu_+ \neq A^\mu_-$ to $g^\pm_{AB}$. The absence of the scale factor when this structure was proposed in [7] set the scale of $\mathcal{H}$ as larger than the scale in either of $\Sigma^\pm$ so the present convention is more natural. A full metrical analysis remains to be carried out.

The method for obtaining the induced $g^\pm_{\mu\nu}(\chi^4_\pm)$ metrics on $\mathcal{A}$ and Ω differs from the above method for obtaining $g_{\mu\nu}$ as a superposition. Part of the future work described in this paper will be to determine the transformations between the abstract and physical coordinates at constant $\chi^4_\pm$. (Such a transformation cannot exist at undefined $\chi^4_\pm = 0$.) Cases for $A^\mu_\pm \neq 0$ should be developed to determine how the condition of maximal symmetry in dS and AdS is perturbed by non-vanishing EM. It is known that static dS or AdS geometry must be supported by a cosmological constant or a constant scalar field so the energy associated with $A^\mu_\pm \neq 0$ should be a main driver of new MCM physics. However, the assumption $A^\mu_\pm = 0$ is useful for describing the model because it equips each slice of constant $\chi^4_\pm$ with a *maximally symmetric* dS_4 or AdS_4 metric. Allowing non-zero $A^\mu_\pm$ will disturb this simplifying condition of maximal symmetry.

In [7], the original statement of the convention for embedding physical metrics on branes located at constant $\chi^4_\pm$ confused the hyperboloid parameter ℓ^2 with the inversely proportional Ricci scalar R so that $\ell^2 = 0$ was associated with $\mathcal{H}$. In fact, $\ell^2 \to \infty$ and $R = 0$ are associated with flatness. This erratum now stands corrected. However, the convention in which $\chi^4_\pm$ is a hyperboloid parameter rather a Ricci scalar suggests a picture of $\chi^4_\pm$ having their origins in $\varnothing$ rather than $\mathcal{H}$ so that $\chi^4_\pm = 0$ defines a topological singularity of infinite curvature due to $\ell^2 = 0$. In later sections, we will show that it is useful to think of $\varnothing$ as a black hole.

Now we will describe the labeled worldsheets of the unit cell. Anticipating an application in which these sheets function as string theoretical Dirichlet branes (D-branes) and referring to the picture of worldsheets as *membranes* arranged in a bulk, we will call these objects **branes**.

- $\mathcal{H}$ is 4D Minkowski space $\mathcal{M}_4$ charted in x^μ. Up to a topological issue of global closure or openness, Minkowski space is the low curvature limit of de Sitter space and/or anti-de Sitter space. $\mathcal{H}$, also called "the $\mathcal{H}$-brane," stitches together $\Sigma^\pm$ at $\lim \chi^4_\pm \to 0^\pm$. Up to a scale factor, $\mathcal{H}$ can be smoothly joined to either of $\Sigma^\pm$. When $A^\mu_\pm = 0$ implies maximally symmetric spacetime in the physical metric at each $\chi^4_\pm$, it is easy to envision a smooth continuum of increasing curvature where $\mathcal{H}$ joins the low curvature limits of dS_4 and AdS_4 at a scale discontinuity. Since $\chi^4_\pm = 0$ is not defined, which follows from $\chi^4_\pm$ being positive- and negative-definite in $\Sigma^\pm$ respectively, $\mathcal{H}$ is a *topological obstruction* between $\Sigma^\pm$. In terms of the open sets of a mathematically formal topological space, no open set can include $\chi^4_\pm = 0$ because it is not defined in the current iteration of the theory. Such topological obstructions are required to separate a pair of Kaluza–Klein theories that double the EM degrees of freedom inherent to a single KKT.

- Ω is a specific worldsheet (the Ω-brane) in Σ^+ located at $\chi^4_+ = \Phi$ where Φ is the golden ratio. In the physical coordinates (with $A^\mu_+ = 0$), Ω is dS_4 with open topology and uniform positive curvature. In Figure 1, Ω spans some width of the horizontal coordinate but that is only meant to demonstrate the spherical geometry of the physical coordinates x^μ_+. Formally, Ω is a single sheet at one value of χ^4_+, as would be $\mathcal{H}$ if $\chi^4_\pm = 0$ was defined.

- $\mathcal{A}$ is a specific worldsheet (the $\mathcal{A}$-brane) in Σ^- located at $\chi^4_- = -\varphi$ where $\varphi = \Phi^{-1}$. In the physical x^μ_- coordinates (with $A^\mu_- = 0$), $\mathcal{A}$ is AdS_4 with closed topology and uniform negative curvature. In Figure 1, $\mathcal{A}$ spans some width of the horizontal coordinate but that is only a representation emphasizing the hyperbolic geometry of the x^μ_- coordinates. Previous work in the MCM has been such that the distance from $\mathcal{H}$ to Ω should be either Φ or Φ^2 times that between $\mathcal{A}$ and $\mathcal{H}$. Setting Ω at $\chi^4_+ = \Phi$, these conventions place $\mathcal{A}$ at $\chi^4_- = -1$ or $\chi^4_- = -\varphi$. Therefore, the abstract distances between $\mathcal{A}$ and $\mathcal{H}$, and between Ω and $\mathcal{H}$ may be revised pending the adoption of another convention.

- $\varnothing$ is an unknown connective element joining Ω and $\mathcal{A}$. It may be a 4D surface or a 5D volume. In general, there is no smooth connection from the Lorentzian $\{-++++\}$ metric in Σ^+ to the pseudo-Lorentzian $\{-+++-\}$ metric in Σ^-. If we take $\varnothing$ to be the worldsheet of a black hole, placement of a singularity at the interface between $\Sigma^\pm$ might help wash out the discrepancy between their topologies. Increasing the curvature of the slices of $\Sigma^\pm$ to the positive and negative infinite limits at $\varnothing$ may also make it easier to join non-vanishing positive and negative curvature on a singularity than it would be to join them on discontinuous but finite positive and negative curvatures. In other words, $R = \pm\infty$ Ricci scalars should be less discontinuous than finite $R_{\mathcal{A}} < 0 < R_{\Omega}$. Placing a black hole at $\varnothing$ should minimize geometric *and* topological discontinuities between Ω and $\mathcal{A}$.

The standard cosmological model (SCM) describes a 4D spacetime: the universe. The SCM is cited as some generalized picture of the Friedmann–Lemaître–Robertson–Walker cosmology, or the more modern ΛCDM model. Either model is more specific than what is required to describe the MCM as an extension of an informally labeled SCM. Indeed, the MCM is more quantum mechanical in nature now than cosmological and the exact details of an underlying standard cosmology, an equation of state for example, are not needed to describe the basic elements.

The main jumping off point for separating the MCM from the SCM was the implementation of a cyclic cosmology [31,39]. Cyclic cosmology is a variant of big bang cosmology that assumes a big crunch at the end of things, and that the crunch serves as a big bang for a new cycle of cosmology. Sometimes it is said that cyclic cosmology is unphysical due to the observed thermodynamic state of the universe but such issues can be sidestepped in a number of ways. The Borde–Guth–Vilenkin theorem [56] which claims to rule out an infinite timelike parameter in the past, which is required for infinite cyclic cosmology, is discussed in Section 45. Another argument claims that it is unphysical to identify the high entropy final state of one cosmology cycle with the low entropy initial state of an identical cycle but the MCM is such that two universes converge on each bounce, one in forward time and one in negative time [31]. When the thermodynamic arrow of time points oppositely in each universe, the increment of entropy at the conclusion of one universe's cycle is offset by the decrement of entropy in the other universe. Furthermore, there is little reason to think that cosmology is so well understood that theoretical arguments might categorically rule out exotic behaviors on cosmological time scales. Beyond that, the present incarnation of the MCM is not necessarily a model of big bang cosmology in any guise at all because the periodicity assigned at first to x^0 has been reimplemented along χ^4. This writer considers it an open question whether or not the MCM in its current incarnation is a model of big bang cosmology in any form. In other words, it is not yet determined whether the added periodicity in χ^4 has replaced the previously supposed x^0 periodicity, or if it has complemented it. In the absence of cyclic cosmology, eternal cosmology is a viable alternative.

In the original MCM language [31, 39], big bangs and big crunches were replaced with *big bounces*. Bouncing is a periodicity in the x^0 direction: vertical on the page of Figure 1. This writer was introduced to cyclic cosmology via loop quantum cosmology (LQC) [57] but the first iteration of the MCM [31] contained nothing specific to LQC which is not found in all other models of cyclic cosmology. For the present version of the MCM unit cell, the main modification to the SCM is the fifth embedding dimension χ^4. It was added a few years after the 2009 publication of the paper which gives the MCM its name [31]: "Modified Spacetime Geometry Addresses Dark Energy, Penrose's Entropy Dilemma, Baryon Asymmetry, Inflation and Matter Anisotropy."

The fifth dimension was implemented following a review of Kaluza–Klein theory. Overduin and Wesson write the following [8].

> "Kaluza's achievement was to show that five-dimensional general relativity contains both Einstein's four-dimensional theory of gravity and Maxwell's theory of electromagnetism. He however imposed a somewhat artificial restriction (the cylinder condition) on the coordinates, essentially barring the fifth one a priori from making a direct appearance in the laws of physics. Klein's contribution was to make this restriction less artificial by suggesting a plausible physical basis for it in compactification of the fifth dimension. This idea was enthusiastically received by unified-field theorists, and when the time came to include the strong and weak forces by extending Kaluza's mechanism to higher dimensions, it was assumed that these too would be compact. This line of thinking has led through eleven-dimensional supergravity theories in the 1980s to the current favorite contenders for a possible 'theory of everything,' ten-dimensional superstrings."

Klein supposed that the fifth dimension might not contribute because it is compactified at an unobservably small scale. The MCM unit cell is purposed to motivate the cylinder condition by requiring that observable physics takes place only on surfaces of constant χ^4. Derivatives with respect to the fifth dimension can't contribute in $\mathcal{H}$ due to an effective condition $\chi^4_\pm = 0$. The same holds for Ω and $\mathcal{A}$ at constant $\chi^4_\pm$. All derivatives with respect to a constant vanish.

Another shortcoming of KKT highlighted by Overduin and Wesson [8]—the main one which prevented the success of KKT in its effort to unify gravitation with classical electromagnetism—is that the only allowable solutions require a vanishing electromagnetic strength tensor $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. It is hoped that doubling the number of EM degrees of freedom from four as in

$$g_{AB} = \begin{pmatrix} g_{\mu\nu} + \phi^2 A_\mu A_\nu & \phi^2 A_\mu \\ \phi^2 A_\nu & \phi^2 \end{pmatrix} \quad , \quad \text{with} \qquad A^\mu = (A^0, A^1, A^2, A^3) \quad ,$$

to eight as in

$$g^\pm_{AB} = \begin{pmatrix} g^\pm_{\mu\nu} + f(\chi^4_\pm) A^\pm_\mu A^\pm_\nu & f(\chi^4_\pm) A^\pm_\mu \\ f(\chi^4_\pm) A^\pm_\nu & f(\chi^4_\pm) \end{pmatrix} \quad , \quad \text{with} \qquad A^\mu_\pm = (A^0_\pm, A^1_\pm, A^2_\pm, A^3_\pm) \quad ,$$

will provide a workaround by which $F_{\mu\nu} \neq 0$ solutions can be extracted in $\mathcal{H}$ from two disconnected Kaluza–Klein theories in $\Sigma^\pm$.

The MCM unit cell reflects the ground state condition in which $A^\mu_\pm = 0$ but it is expected that the non-zero $A^\mu_\pm$ solutions can be implemented as perturbations or more complicated exact solutions. The result for $A^\mu_\pm \neq 0$ will be that the $\mathcal{H}$-, $\mathcal{A}$-, and Ω-branes lose their shared character of *maximal* symmetry. In the $A^\pm_\mu = 0$ ground state, the piecewise fifth dimension $\chi^4_\pm$ charts a continuum of increasingly curved, maximally symmetric physical spacetimes between $\mathcal{A}$ and Ω disrupted only by a scale discontinuity at $\mathcal{H}$. This serves as a toy model upon which one would build more realistic applications. To make use of the expanded degrees of EM freedom in $\Sigma^\pm$, one must use $A^\mu_\pm$ to define A^μ in $\mathcal{H}$. This is implemented by a mechanism well known from classical EM: A^μ is taken as a function of the advanced and retarded potentials $A^\mu_{\text{adv}} = A^\mu_+$ and $A^\mu_{\text{ret}} = A^\mu_-$ [7]:

$$A^\mu = c_+ A^\mu_+ + c_- A^\mu_- \quad .$$

The idea to have the physics of the observable universe ($\mathcal{H}$) defined by two 5D theories reflects a principle called holographic duality. This idea was made famous by Maldacena's demonstration of a "correspondence" between a 4D conformal field theory and AdS_5 [58]. The MCM flavor of "holographic duality" between the physics of a 4D surface and two adjoining 5D bulks is simpler than Maldacena's famous AdS/CFT duality but the duality is *holographic* nonetheless. The mechanism reflects exciting new thinking. Usually, holographic duality between a surface and a bulk is considered to be such that the surface is the exterior boundary of one simply-connected bulk. The *fresh new idea* for holographic duality in the MCM is to sandwich a holographic surface between two bulks. This idea alone far separates the MCM from competing theories. It cannot be overstated that the MCM has accomplished what other theories have not accomplished due in large part to this original thinking in the red-hot area of bulk-boundary physics. Although this writer was not acquainted with Randall–Sundrum models (Section 42) when constructing the unit cell, it is quite like a third class of RS model not considered by Randall and Sundrum. The two famous RS1 and RS2 models put branes at one side of a bulk or another—at infinity, finite distance, or zero in their given coordinates—but they do not consider the case of a brane set between two asymmetric bulks.

Before continuing on to the MCM particle scheme (Section 0.3), the reader's attention to called to the reality that certain labeling conventions in the unit cell are chosen intuitively from among a few possible permutations. The purpose in this program is to facilitate easy discussion that would be clouded by repeated clarifications for caveats about all possible permutations. Usually, the number of possible permutations is low and the alternatives reflect little more than a sign change. For instance, the assignment of dS or AdS geometry to slices of constant $\chi^4_\pm \in \Sigma^\pm$ is only a sign convention. It is assumed that the trip from $\mathcal{H}_1$ to $\mathcal{H}_2$ goes through Ω, and then $\mathcal{A}$, but this is subject to reversal if needed. For instance, the cosmological constant in AdS is negative while that in dS is positive. The energy landscape might override the assumed convention. The fifth dimension is currently timelike in Σ^- and spacelike in Σ^+ but if the opposite convention were desired, one would add a minus sign into the metric. If we move the origins of $\chi^4_\pm$ to $\varnothing$, then the natural sign conventions for $\chi^4_\pm$ would be reversed, etc. In the end, we will require that binding energy is negative in $\mathcal{H}$ and that the entropy in $\mathcal{H}$ tends to increase with increasing x^0. Everything else should be arranged accordingly.

In addition to the geometric objects labeled in Figure 1 and detailed above, there are some algebraic objects of fundamental importance. We introduce new algebraic complexity by attaching different state spaces to the various labeled manifolds. For instance, $L^2(\mathbb{R}^3)$ is the well known Hilbert space of square integrable functions of three real variables. L^2 describes the algebraic state space and $\mathbb{R}^3$ describes the domain of the wavefunctions which are representations of the L^2 states. Using $\mathcal{H}' \equiv L^2(\mathbb{R}^3)$ to denote the space of position states in $\mathcal{H}$, $\mathbb{R}^3$ refers to the 3D spatial submanifolds of $\mathcal{H}$ described by the $\{+++\}$ part of $\mathcal{M}_4$'s $\{-+++\}$ signature. We will use $\mathcal{A}'$ and Ω' to label the state spaces of particles located on the $\mathcal{A}$- and Ω-branes. Although the wavefunctions of states in $\mathcal{A}$ and Ω are also functions of three real variables, those variables do not chart the 3-space in the Euclidean metric δ_{ij} that is usually inferred from the $\mathbb{R}^3$ symbol. Formally, $\mathbb{R}^3$ is any tuple of three real variables and E^3 is Euclidean 3-space. These two symbols are often intermingled in physics where E^3 may be less familiar. Therefore, increased nuance is warranted for the labeling.

With $\mathcal{A}$ as AdS_4 and Ω as dS_4, the domains of the functions in the $\mathcal{A}'$ and Ω' state spaces are hyperbolic H^3 and spherical S^3 respectively.[1] We might write, for example, $\Omega' \equiv L^2(S^3)$ to indicate that the $\mathbb{R}^3$ coordinates in the domain of wavefunctions in Ω are not subject to the Euclidean metric as are wavefunctions in $\mathcal{H}$ with $\mathcal{H}' \equiv L^2(E^3)$.

For reasons developed below, mainly to accommodate the eigenstates of observable operators with continuous spectra such as $\hat{x}$, we will introduce rigged Hilbert space to employ other algebraic spaces than L^2 for position states located in various sectors of the unit cell. Readers unfamiliar with rigged Hilbert space are referred to [59–61]. In the following, we omit some nuance differentiating state spaces and function spaces.[2]

- $\{\mathcal{H}', \mathcal{A}', \Omega'\}$ is a rigged Hilbert space (RHS), also called a Gelfand triple. $\mathcal{H}'$ is a subspace of $\mathcal{A}'$. Ω' is a dual (or antidual) space to $\mathcal{H}'$ which contains $\mathcal{A}'$ as a subspace: $\{S_1, S_2, S_3\}$ such that $S_1 \subset S_2 \subset S_3$. In previous work, we have used the convention that RHS is $\{\mathcal{A}', \mathcal{H}', \Omega'\}$ but the structure of RHS suggests that S_1 is most appropriate for the manifold of physical observables [61]. That manifold is $\mathcal{H}$ so we have chosen the present convention for $\{\mathcal{H}', \mathcal{A}', \Omega'\}$. The previous convention in which the order of the spaces in the triple matched the order of the branes in the unit cell was intuitive but it does not appear to be the one supported by the definitions.

- $\mathcal{A}'$ is Hilbert space. In this book, the relevant Hilbert space is usually taken as the infinite dimensional Hilbert space of position states. In that case, $\mathcal{A}'$ is the $L^2(\mathbb{R}^3)$ space of square integrable functions: wavepackets rather than the δ function position eigenstates. One might write this as $L^2(H^3)$ to indicate that the domain of these L^2 wavefunctions possesses hyperbolic geometry.

- $\mathcal{H}'$ is a subdomain of Hilbert space $\mathcal{H}' \subset \mathcal{A}'$. Under certain conditions related to unbounded observable operators with continuous spectra such as the position operator $\hat{x}$, there exist states in $\mathcal{A}' \equiv L^2$ for which certain ordinary quantum mechanical identities fail. $\mathcal{H}'$ is the subdomain of $\mathcal{A}'$ in which things like the expectation value and uncertainty formulae are guaranteed to be well behaved for every state in the space. De la Madrid presents these details in [59–61]. Due to the stated properties of well behavior, the S_1 part of an RHS $\{S_1, S_2, S_3\}$ is attached to the 4D physical universe of observables: $\mathcal{H}$. The present convention contrasts the previous convention in which S_2 was attached to $\mathcal{H}$.

[1] AdS_3 and dS_3 refer to Lorentzian manifolds, meaning that these are not the spatial parts of AdS_4 and dS_4. Rather, AdS_3 and dS_3 are manifolds spanned by one timelike dimension and two spacelike dimensions.

[2] Ballentine writes [62], "It is a matter of taste whether one says that the set of functions forms a representation of the vector space, or that the vector space consists of the functions $\psi(\mathbf{x})$."

- Ω' is the dual (or antidual) space of $\mathcal{H}'$ such that $\{\mathcal{H}', \mathcal{A}', \Omega'\}$ is an RHS. Eigenstates of operators with continuous spectra are non-normalizable Dirac δ functions which do not exist in $\mathcal{A}'$ or $\mathcal{H}'$. Such eigenstates, usually position eigenstates, belong to the state space Ω' satisfying $\mathcal{H}' \subset \mathcal{A}' \subset \Omega'$. As will be discussed in Section 1, predictions for what will happen in the future reside in Ω. Since the MCM seeks to restore a classical character of motion which was lost in quantum mechanics, meaning that a prediction for a time-advanced quantum position state should be a point in spacetime as was the case for classical motion, the S_3 part of the RHS $\{S_1, S_2, S_3\}$ containing Dirac δ wavefunctions is assigned to Ω and called Ω'.

- $\varnothing'$ is a hypothetical state space for states in the $\varnothing$-brane.

0.3 The MCM Particle Scheme

Early work in the MCM [31] posed a solution to the mystery of the matter asymmetry. That mystery regards why the universe is made of matter rather than anti-matter [63]. The issue is similar to a question about non-conservation of 4-momentum at the big bang. If nature is thought to conserve baryon number and 4-momentum, then why should the big bang not conserve both?[1] It was suggested in [31] that two universes leaving a big bang, or a big bounce, should be understood as an ordinary particle pair in the sense of pair creation by vacuum fluctuations. It is not known why any particular fluctuation occurs but the particle production process is better understood than an alleged cosmological big bang process for a single universe with an anomalous increment of momentum and an anomalous baryon number. In the particle pair picture, the forward and reverse time universes are a particle and an anti-particle. One has positive baryon number and positive p^0. The other has negative baryon number and negative p^0. The MCM model of particles [6] follows from this notion: a universe, one quantum of MCM spacetime, is like a fundamental matter particle.

In the unit cell, our observable universe given positive baryon number B is the $\mathcal{H}$-brane. It is spanned by x^0 and x^i. The MCM particle scheme supposes that all fundamental matter particles are quanta of spacetime spanned by a spatial unit vector $\hat{x}^i$ and a temporal one: $\hat{x}^0$ or $\hat{\chi}^4_\pm$. Given these two types of time in the MCM, chronological x^0 and chirological χ^4, this thinking leads to the 12 well known members of the three generations of matter particles.

[1]Positive baryon number is associated with matter and negative baryon number is associated with anti-matter. For historical reasons [63], the excess of matter over anti-matter is described as an excess of baryons over anti-baryons despite there being a similar excess of leptons over anti-leptons.

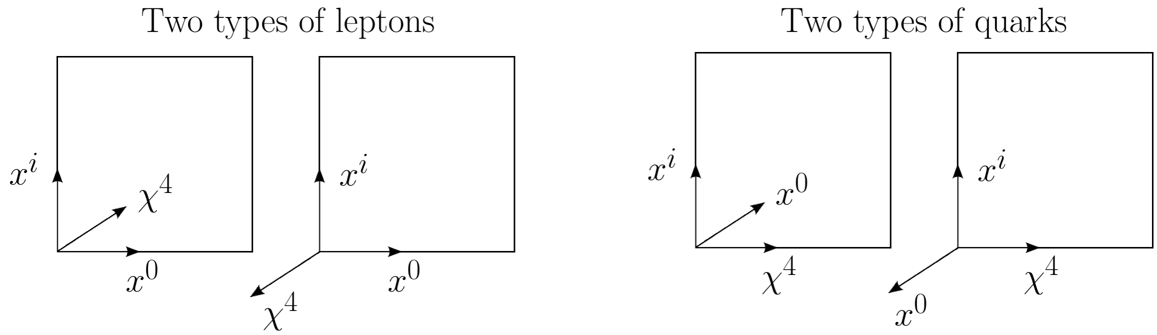

Figure 2: It is supposed that the fundamental matter particles of the standard model represent geometric quanta in the MCM unit cell. Leptons are planes formed by x^i and x^0 while quarks are planes formed from x^i and χ^4. Two varieties of each are formed when the unused instance of x^0 or χ^4 forms a right- or left-handed orthogonal triad. We will associate the three color charges of QCD with the $\{\chi^4_+, \chi^4_-, \chi^4_\varnothing\}$ varieties of chiros.

Referring to Figure 1, space x^i points into the page. Chronos points up and chiros points to the right.[1] The spanning bases for planar spacetimes are x^0x^i and χ^4x^i. The basis vectors in the respective directions can form left- or right-handed coordinate systems with the third member of $\{x^0, x^i, \chi^4\}$ so there exist four distinct varieties of MCM spacetime quanta: space crossed with either of chronos or chiros, each in left- and right-handed varieties, as in Figure 2. The planes of x^i crossed with the well studied x^0 flavor of time are taken as the relatively well-behaved leptons. Space crossed with the exotic new chirological time is taken as a quark. We suggest that quantum electrodynamics (QED) is simple relative to quantum chromodynamics (QCD) because x^0 is simple relative to $\chi^4 \cong \{\chi^4_+, \chi^4_\varnothing, \chi^4_-\}$. The three color flavors of each quark are distinguished by the three varieties of χ^4. We say quarks are never observed in isolation because the piecewise structure of χ^4 is such that $\chi^4_\pm$ are each needed to construct an instance of the unit cell. The existence of $\Sigma^\pm$ implies the coexistence of $\Sigma^\mp$.

Having established two leptons and two quarks (Figure 2), the three generations of each are associated with the $\mathcal{H}'$, $\mathcal{A}'$, and Ω' state spaces, as in Figure 3. In the final analysis, the primary distinction among the three generations may be attributed most directly to the three different lattice positions $\{\mathcal{A}, \mathcal{H}, \Omega\}$, or to the three different state spaces $\{\mathcal{H}', \mathcal{A}', \Omega'\}$. The three generations of matter particles reflect the structure of the unit cell but the details of the MCM state spaces are not finalized. Thus, it cannot be determined at this time if the three generations of particles follow

[1] In Greek, chronos and chiros refer to "man's time" and "gods' time" respectively.

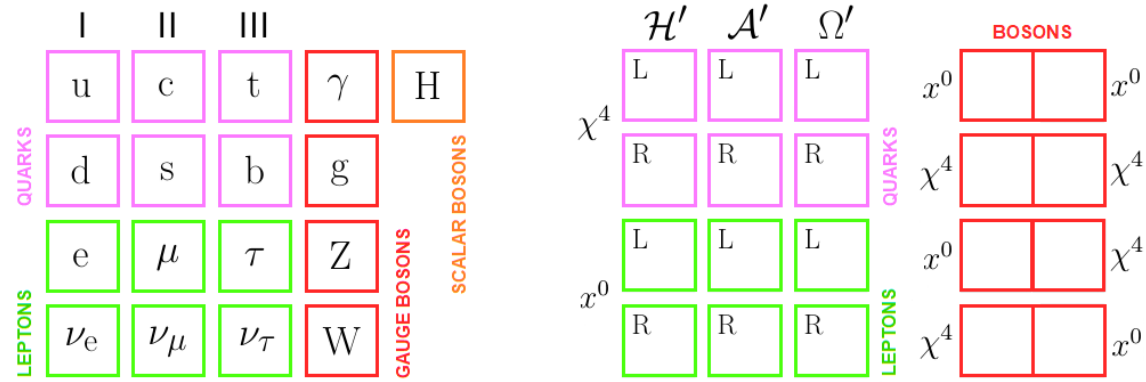

Figure 3: The MCM particle model (right) compared to the standard model of particle physics (left). Each instance of x^0 or χ^4 refers to a spacetime spanned by x^i and either x^0 or χ^4. The scalar Higgs boson is an outlier in the standard model. There are are no such outliers in the modified model.

more directly from algebraic distinctions among $\{\mathcal{A}', \mathcal{H}', \Omega'\}$ or geometric distinctions among $\{\mathcal{A}, \mathcal{H}, \Omega\}$. Presently, the three generations of leptons and quarks are increasingly massive and we would like to associate this property with the $\mathcal{H}' \subset \mathcal{A}' \subset \Omega'$ structure of RHS. Since electrons are stable in $\mathcal{H}$ while muons and taus are not, this suggests the convention in Figure 3: $\mathcal{H}'$ should be the state space corresponding to the first, lightest generation of matter particles. Associating increasing mass with increasing scale factor across the unit cell centered on $\varnothing$ would suggest Ω for the second generation particles. Perhaps the three generations of matter particles observable in $\mathcal{H}$ would be better associated with Ω, $\varnothing$, and $\mathcal{A}$ in an alternative, similar convention. Most importantly, it is emphasized that the permutations of the unit cell match the permutations of the particles.

Another consideration for the MCM state space structure regards lepton universality. The standard model predicts that each lepton flavor should be identical to the others up to its mass. However, modern experiments suggest that this is not the case. The proton radius puzzle observed in the muonic hydrogen system [64] is an example of experimentally determined non-universality among lepton flavors. By putting each of the MCM matter particle generations into a different state space, we motivate lepton non-universality in principle, as required for agreement with experiment.

We have relied to some degree on phenomenological considerations when constructing this model of particles. Still, the model suffices to claim a first principles derivation of the particle spectrum. The unit cell has permutations of its objects generating two pairs of particles in three varieties. One of those pairs of particles may be distinguished by three further varieties of QCD color charge with $\{+,-,\varnothing\}$. The fundamental bosons are well accommodated too. It is known that the 12 fundamental matter particles are spin-1/2 fermions so we assign that property to each MCM quantum of spacetime by supposition. Spin-1/2 is well aligned with $\chi^4_\pm$ spanning only one of Σ^+ or Σ^-, but never both. Similarly, the scale of any MCM spacetime quantum will be half the width of the unit cell. The force carrying particles of the standard model are known to have spin-1 so the MCM bosons are assembled from pairs of matter particles. This is done in part because $\frac{1}{2}+\frac{1}{2}=1$ and in part because forces are usually transmitted between pairs of fermionic matter particles.

Being the most ordinary and well understood force carrying particle, the photon is the x^0x^0 particle at the top of Figure 3's stack of elementary MCM bosons. The most complicated, least understood elementary boson is the gluon g associated with the $\chi^4\chi^4$ connection. In this arrangement, we find more support for the MCM particle scheme. It is known from experiment that there exist eight varieties of gluon. A triumph of the MCM is that we obtain eight such varieties in the unit cell. Quark flavor is associated with the three varieties of χ^4. Gluons are associated with connections between quarks. The nine permutations of a $\chi^4\chi^4$ connection are $++$, $+\varnothing$, $+-$, $\varnothing+$, $\varnothing\varnothing$, $\varnothing-$, $-+$, $-\varnothing$, and $--$. Removing $\varnothing\varnothing$ on some qualitative grounds (which may be inferred from the $\varnothing$ symbol itself), we are left with eight varieties of gluon.

Why should $\varnothing\varnothing$ not be associated with a gluon? There are many possible reasons but it is hoped that the reason will fall out from future inquiry. Since $\chi^4_\varnothing$ has no length in the convention where Ω is joined to $\mathcal{A}$ by a single point, the $\varnothing\varnothing$ gluon has no moment, in some sense. The other eight connections do have non-vanishing moments, in that sense. Another reason might be that the other eight gluons connect to $\mathcal{H}$ through $\Sigma^\pm$ while $\varnothing\varnothing$ does not. For that reason, it may not be observable, or may not be directly observable. As we will detail in Section 1, all observations are necessarily made in $\mathcal{H}$ so the property of being observable may depend on connection to $\mathcal{H}$. Another possibility is that there are, indeed, nine gluons, and that a nine gluon model would improve the theory of QCD. One might take the $\varnothing\varnothing$ connection as a sterile gluon in the manner that sterile neutrinos are sometimes thought to exist. In general, the total picture of QCD physics is complicated and has a lot of room for improvement.

Ignoring a hypothetical Higgs boson, the only remaining standard model particles requiring placement in the modified model are the W and Z bosons. These are accommodated by either of the two remaining connections: $x^0\chi^4$ or χ^4x^0. Choosing the former, the original assignment in [6] cast $W^\pm$ as $x^0\chi^4_\pm$ and Z^0 as $x^0\chi^4_\varnothing$ [6]. It is emphasized that the unit cell's permutations' *multiple exact likenesses* to experimentally determined particle properties are evidence that the MCM is a good theory. The weak force governs interactions between leptons and particles made of quarks so, therefore, the admixture of the x^0 and χ^4 elementary fermions in the $x^0\chi^4$ weak boson connection is philosophically robust and physically sound.

We have randomly chosen the $x^0\chi^4$ connection for W and Z. We might have chosen χ^4x^0. In either case, the MCM predicts at least one more spin-1 elementary particle, possibly three, in the remaining partner to $x^0\chi^4$ or χ^4x^0, as in Figure 3. However, there exists another theoretical variant which was not mentioned in the first iteration of the MCM particle scheme [6]. We have associated the $W^\pm$ particle/anti-particle pair with $\chi^4_\pm$ while we have not placed anti-gluons in the $\chi^4\chi^4$ connection. If the $\pm$ scripting does not specify the anti-particle for gluons, then neither should it for W. Therefore, we might (should) associate the Z and W particles with only two of $\{x^0\chi^4_+, x^0\chi^4_-, x^0\chi^4_\varnothing\}$. In that case, we would suppose that the Higgslike particle is the third member of the $x^0\chi^4$ connection, that $x^0\chi^4$ and χ^4x^0 are indistinguishable, and that the Higgslike spin-1 particle completes the smorgasbord.

Whatever the exact details are, the modified model predicts that there should be no spin-0 fundamental particles. Therefore, the Higgslike particle must have spin-1. If the Higgslike particle is eventually determined to have spin-1, that will be strong evidence that time and effort should be invested in the theses given in the remainder of this book.

Part I: The Modified Cosmological Model

1 The $\hat{M}^3$ Operator and its Equation

While it is standard in physics communications to put main results at the beginning and then explain them, this will not be possible for $\hat{M}^3$. Without developing the context first, the main results could not be conveyed well. Therefore, Sections 1.2 through 1.7 will mostly lay the foundation for more interesting results in Sections 1.8 through 1.11.

1.1 Introduction

The fundamental equation of classical mechanics $\mathbf{F} = \partial_t\mathbf{p} = m\partial_t^2\mathbf{x}$ is postulated in Newton's laws. The fundamental equation of quantum mechanics, $i\hbar\partial_t\psi = \hat{H}\psi$, is usually implemented as a postulate. In both cases, the differential operators ∂_t and ∂_t^2 (or the ∂_x^2 in $\hat{H}$) are used in postulated equations. In the MCM, we would like to obtain a new equation for $\hat{M}^3 \propto \partial_t^3$ such that the discrepancies between classical reality and quantum theory are lessened or remedied. Various postulates or hypotheses for the functioning of $\hat{M}^3$ have appeared in earlier MCM publications and, indeed, the number of variations approaches the number of papers written about them. In the end, the postulate should be the only expression consistent with the requirements, up to the form of the representation. At that time, putting the correct equation to paper should be effortless. For this reason, previous work in the MCM has more closely attended that which $\hat{M}^3$ needs to do than the formal statement and study of a postulate like $F = m\ddot{x}$ or $i\hbar\dot{\psi} = \hat{H}\psi$. In this long section, we will examine the $\hat{M}^3$ operator which has been identified as an appropriate operator for what should be some new equation for a theory of everything.

1.1.1 The $\hat{M}^3$ Operator

$\hat{M}^3$ describes the actions of a physicist. Although the extant quantum theory requires a physicist's actions to implement wavefunction collapse upon measurement, the usual approach to quantum mechanics (QM) ignores the rest of what the physicist does. In efforts to better understand quantum theory, epistemological considerations sometimes fixate on an artificial distinction between a quantum state and an ideal measuring apparatus. It is asked how an *ideal* measurement can be made when detectors are necessarily quantum mechanical themselves. Compounding such questions, many experiments such as the double-slit and delayed-choice quantum eraser experiments [65] show that observation is supremely weird within the existing framework. The main new idea in the MCM seeks to separate the physicist from his experiment rather than to separate a hypothetical ideal detector from its quantum subject matter. Measurement is made ideal as a psychological process divorced from anything manifestly quantum mechanical. It is hoped that the description of a time-evolving quantum state will be more natural in this framework.

Regarding questions of epistemology that don't impede one's ability to compare experiments to predictions, physics may be differentiated between work in the esoteric fundamentals and work in the more glamorous applications [66]. The latter is less concerned with philosophical problems but the MCM is a program in the subbasement of the fundamentals. We ask questions such as the following. Is it a step too far to suppose that there exists a better framework? Perhaps there is one to which the current theory is only an approximation? Is it wrong not to shut up and calculate? To these ends, we have identified $\hat{M}^3$ as a good operator for what should be a new revolution in the arena of the fundamentals.

The psychological process for $\hat{M}^3$ was defined as follows [3].

> "To test any theory[,] two measurements must be made. Call these measurements A and B corresponding to events a and b. The boundary condition set by A will be used to predict the state at b. To make this prediction[,] the observer applies physical theory to trace a trajectory from A to the future event b. Before the observer can verify the theory, sufficient time must pass that the future event occurs. Once this happens[,] a retarded signal from b reaches the observer in the present and a second measurement B becomes possible. [F]rom the present[,] the observer traces a path into the future. Once that future becomes part of the observer's past, a signal reaches the observer in the present and the theory can be tested. A three-fold process.
>
> $$\text{Present} \mapsto \text{Future} \mapsto \text{Past} \mapsto \text{Present}^1 \text{ ."} \tag{1.1.1}$$

The process of $\hat{M}^3$ starts at A. Some event a has already occurred. The signal from a has reached the observer who has represented the condition of a as some abstract or analytical expression. For instance, a detector has registered a particle at some point in space, or in some region of spacetime,[2] and then the detector told the observer what it saw. The observer says, "Given my observation A, I predict by theoretical construction that a subsequent event b will occur, which I will observe at B." This prediction is the first step of $\hat{M}^3$. It is an abstract prediction Present $\mapsto$ Future. The next step requires a time translation of the observer to some time later than the time associated with the predicted event. Since we expect $\hat{M}^3$ to operate on states rather than the observer, the observer's time translation might be implemented as a translation of a and/or b to an earlier time. This is the second step Future $\mapsto$ Past. The third step is a reconnection to the psychological level when the signal from b comes to the observer's attention at B: Past $\mapsto$ Present. It is hoped that a new equation which reflects this process will improve quantum theory and human understanding. Feynman states the idea in [67].

> "[T]here is always hope that [a] new point of view will inspire an idea for the modification of present theories, a modification necessary to encompass present experiments."

[1]The $\mapsto$ symbol was chosen only so as to use a generalized arrow symbol for this word-level expression.

[2]Whether an apparatus detects the particle at a point or merely within some region is an interesting and open question. In the end, all that is known is that the observer cannot glean more information from the apparatus than the region of spacetime in which the particle is detected.

1.1.2 Principles and Equations

Einstein's greatest genius was to conceive of the equivalence principle. Briefly, experiments done in gravity must yield the same results when done in a spaceship under the same acceleration. To formalize his principle mathematically, Einstein had to collaborate for several years with mathematicians such as Grossmann but Einstein's true genius was not finding Einstein's equation. The work of profound genius was to conceive of a new principle which must be satisfied by an equation *in some form*. Finding that equation, while difficult and admirable, was ultimately a labor. Einstein describes himself as working "like a horse" in his quest to find the equation once the principle was set. Similarly, Newton was in correspondence with Leibniz to some degree during the development of calculus but Newton is regarded as the supreme genius due to his conception of the laws of motion. The modern mathematical statement of classical mechanics is mostly due to Cauchy, not Newton, but Newton is regarded as the grandfather of physics because the highest achievement is the formulation of new principles. As the laws of motion must be satisfied, and as the equivalence principle must be satisfied, the MCM process for $\hat{M}^3$ must be satisfied. The description of the three-fold psychological process for $\hat{M}^3$ is as irrefutable and self-evident as any other principle in physics. There must exist a mathematical language for describing it.

1.1.3 Targeted Issues in Quantum Theory

After introducing notation in Section 1.2 for associating quantum states with the elements of the unit cell, we will present cases that $\hat{M}^3$ should be useful for the following.

- To implement dynamical rather than ad hoc wavefunction collapse (Section 1.8).
- To explain the origin of the fine structure constant (Section 1.9).
- To promote the metric from a disconnected background in quantum theory to a dynamical object in it via a new theory of quantum gravity (Section 1.10).
- To find use cases in physics for new mathematical tools related to fractional distance analysis (Section 1.6) [2], and to do a few other things.

The usual formulation of quantum theory provides no dynamical mechanism for wavefunction collapse, also called state reduction or projection. With $\hat{M}^3$, the MCM adds some extra steps to time evolution that are purposed to accommodate such a mechanism. Presently, collapse is inserted into QM as needed to force agreement with experiment. If dynamical collapse is achieved, quantum theory will be much improved. Isham writes the following regarding this most glaring gap begging for improvement [68].

> "[T]he idea of a reduction of the state vector is often invoked in more realist approaches in which the state vector is deemed to refer to a single system. The reduction is then assumed to occur after a *single* (ideal) measurement, and has nothing to do with system selection in a series of repeated measurements. From this perspective, the overall time development of a state of a single system consists of sharp jumps produced by the act of measurement, separated by periods of deterministic evolution governed by the Schrödinger equation[.]
>
> "The major problem is to understand the origin of these sudden changes in the state. In particular, can they be obtained from the existing quantum formalism, or does the reduction of the state vector have to be added to the general rules of quantum theory as a fundamental postulate? This problem is particularly acute in any approach to quantum theory that aspires to demote 'measurement' from playing a fundamental part in the formulation of the theory. In this case, there is a strong motivation to try to derive the state reduction vector from the existing formalism; albeit, perhaps, only as an empirically useful approximation to the actual development of the state in time.
>
> "The nature of the problem depends in part on the perceived referent of the state. If the state is held to quantify our *knowledge* of the system, then the reduction process is arguably analogous to the conditioning procedure in classical probability in which the addition of extra information about what is actually the case changes our state of knowledge. On the other hand, if the state vector is held to refer to the system itself, then the idea of reduction is frequently tied to the 'uncontrollable disturbance' thesis. This raises the obvious question of the possibility of understanding the nature of this effect in direct physical terms. In particular, what type of interaction serves as an 'ideal measurement'?
>
> "One approach to this problem is to ask again about the significance of the fact that actual measuring devices are made of quantum atoms. Is it possible to understand a state reduction as the outcome of some dynamical evolution in which object and apparatus are both regarded as quantum-mechanical systems? Indeed, even within the minimal, pragmatic approach to quantum theory there is good reason for asking what *type* of interaction

> between two systems is to be regarded as a *bona fide* measurement of one by the other. The concept of measurement plays a fundamental role in the formulation of quantum theory, and therefore deserves to be understood further."

Measurement is of fundamental importance in the MCM. Each measurement of a quantum system corresponds to an $\mathcal{H}$-brane. Diffusion under the Schrödinger equation happens in the bulk spaces $\Sigma^{\pm}$ and the sharp jump to a collapsed state is associated with $\mathcal{H}_k$. The act of measurement is made ideal as an interaction between a system made of atoms and an observer's non-quantum consciousness.

It remains hard to motivate the value for the MCM fine structure constant α_{MCM} so we will not phrase the present problem of $\hat{M}^3$ in terms of the original motivation [30]. Instead, we will lay out the current best understanding of $\hat{M}^3$ and some problems which are found to deserve further development. Appendix A describes the original program by which the fine structure constant was found and then the existence of $\hat{M}^3$ was deduced from the analytical structure of

$$\alpha_{\text{MCM}}^{-1} = 2\pi + \big(\Phi\pi\big) \approx \alpha_{\text{QED}}^{-1} \quad . \tag{1.1.2}$$

Regarding our intention to supplement the existing framework of quantum theory with $\hat{M}^3$, Finkelstein writes the following [69].

> "Quantum theory began with ad hoc regularization prescriptions of Planck and Bohr to fit the weird behavior of the electromagnetic field and the nuclear atom[,] and to handle infinities that blocked earlier theories. In 1924[,] Heisenberg discovered that one small change in algebra did both naturally."[1]

Heisenberg stated the following in his 1933 Nobel address.

> "Quantum mechanics [*sic*] arose, in its formal content, from the endeavor to expand Bohr's principle of correspondence to a complete mathematical scheme by refining his assertions."

Similarly, it remains to expand the MCM principles to a complete mathematical scheme by refining the assertions about $\hat{M}^3$. To wit, we have found a value α_{MCM}

[1] Heisenberg's famous $\hat{p}\hat{q}-\hat{q}\hat{p}\neq 0$ quantum algebra was a small change in notation but it reflects a giant leap in the ability of humans to understand the natural world. After all, the idea that 3×2 under certain circumstances might not equal 2×3 was a radical departure from thousands of years of previous mathematical thinking. Heisenberg's change of algebraic structure is the origin of the phrase "a quantum leap" meaning "a huge or sudden increase or advance of something."

that falls out of some (mostly) standard quantum mechanical language but we have neither connected that language to the full quantum theory nor explained the 0.4% discrepancy with α_{QED} (Section 1.9.4). There exists an idea for how state reduction might be implemented more naturally in the MCM than it is in QM (Section 1.8) [70] but we have not written down any Eureka-level equations of motion. While such deficiencies remain to be remedied in the course of the work described in this book, the new object $\widehat{\infty}$ called algebraic infinity [2] is most certainly a Eureka-level idea for handling certain infinities that block *current* theories.

On the problem of quantum gravity, we say it is a hard problem because there does not exist a robust mathematical language in which the objects of the gravitational theory can be put into an equation with the objects of the quantum theory.[1] General relativity (GR) is a theory of points in spacetime but the state of being located at a point cannot be measured and does not exist in Hilbert space. Quantum states are fuzzy but GR does not admit fuzziness. Far removed from a theory of gravitons or questions about the curvature of spacetime as a disconnected background to quantum theory, the general problem of quantum gravity is that there does not exist a good framework in which it is possible to put the equivalence relation = between two separate statements of gravitation and quantization. For instance, the equivalence of the inertial mass in classical mechanics and electrodynamics allows us to combine the Lorentz force law with arbitrary mechanical forces. On the other hand, there is no Schrödinger equation for the metric and there is no way to put a probability amplitude into a stress-energy tensor such that it is mutually dynamical with Schrödinger evolution. The MCM mechanism for quantum gravity offers an original and exciting mathematical language in which quantum objects might interact with gravitational objects. However, it very much remains to establish this new language as a complete mathematical framework.

1.2 The Ontological Basis

The process

$$\text{Present} \mapsto \text{Future} \mapsto \text{Past} \mapsto \text{Present} \quad , \tag{1.2.1}$$

[1] There is some machinery in QFT by which a certain tensor field $\varphi_{\mu\nu}$ (called a graviton field) can couple in its two indices to a stress-energy tensor $T^{\mu\nu}$. The QFT graviton can be used to reproduce a few experimental results but most of those come only under a host of simplifications, hand-waving, and cumbersome constraints. The QFT graviton is ugly, not beautiful, and it is useful only for small perturbations on Minkowski space. In the opinion of this writer, furthermore, there is little reason to think that the hypothetical quantum force carrier of the gravitational force is real because there is no gravitational force. Gravitation is geometry in curved spacetime. It is a fact that a rank-2 tensor field can couple to $T^{\mu\nu}$ in QFT but it is not well established that this confluence of tensor indices is well suited to the general problem of quantum gravity. After all, this coupling has been known for decades but there is no consensus on what a working theory of quantum gravity might look like or how one might demonstrate gravitons' existence through observation. Indeed, there is no consensus on the existence of gravitons due in part to the weakness of the theoretical framework for $\varphi_{\mu\nu}$ in applications to gravitation.

is associated with the operator

$$\hat{M}^3 : \mathcal{H}'_1 \to \Omega'_1 \to \mathcal{A}'_2 \to \mathcal{H}'_2 \quad , \tag{1.2.2}$$

and/or its variant

$$\hat{M}^3 : \mathcal{H}_1 \to \Omega_1 \to \mathcal{A}_2 \to \mathcal{H}_2 \quad . \tag{1.2.3}$$

The former describes abstract algebraic translation through rigged Hilbert space. The latter describes geometric translation through coordinate space. $\hat{M}^3$ itself operates on wavefunctions so notation is required to specify where a given wavefunction lives: which of the branes and/or which of the state spaces along the process of $\hat{M}^3$. For instance, we will introduce notation such that a state in $\mathcal{H}'$ has the domain of its wavefunction representation specified as the x^i spatial part of the x^μ physical coordinates charting $\mathcal{H}$. However, if

$$\begin{aligned} \psi \in \mathcal{H}' \quad &\implies \quad \psi = \psi(x^i) \\ \psi \in \mathcal{A}' \quad &\implies \quad \psi = \psi(x^i_-) \\ \psi \in \Omega' \quad &\implies \quad \psi = \psi(x^i_+) \quad ,^1 \end{aligned} \tag{1.2.4}$$

then the $\mathcal{H}' \subset \mathcal{A}' \subset \Omega'$ nested structure of the RHS $\{\mathcal{H}', \mathcal{A}', \Omega'\}$ is superficially confounded. The space of functions of a given variable is not intuitively a subspace of the space of functions of another variable. Still, it is possible that states represented by the former might span a subspace of the states represented by the latter. To avoid any potential problems, an appeal is made to a subtle difference little considered in physics: the difference between state spaces and function spaces. In this section, we will clarify these details somewhat and introduce **the ontological basis**. It assigns wavefunctions to the various branes in the MCM unit cell, and to their corresponding state spaces.

Let $\psi_k : \mathbb{R} \to \mathbb{C}$ be a function and let $\times$ be an inner product. Then

$$\left.\begin{aligned} \mathcal{H}' &= \{\psi_1, \psi_2; \times\} \\ \mathcal{A}' &= \{\psi_1, \psi_2, \psi_3; \times\} \\ \Omega' &= \{\psi_1, \psi_2, \psi_3, \psi_4; \times\} \end{aligned}\right\} \quad \implies \quad \mathcal{H}' \subset \mathcal{A}' \subset \Omega' \quad , \tag{1.2.5}$$

at least approximates an RHS if it does not satisfy the definition directly. To break the nested structure and support an arrangement of functions of different variables,

[1] Recall that $x^\mu_\pm$ are the physical coordinates on slices of $\Sigma^\pm$ at constant $\chi^4_\pm$, as in Section 0.2.

we will append labels as

$$\left.\begin{aligned} \mathcal{H}'_{\texttt{H}} &= \{\psi_1,\psi_2;\times,\texttt{H}\} \\ \mathcal{A}'_{\texttt{A}} &= \{\psi_1,\psi_2,\psi_3;\times,\texttt{Alpha}\} \\ \Omega'_{\texttt{O}} &= \{\psi_1,\psi_2,\psi_3,\psi_4;\times,\texttt{Omega}\} \end{aligned}\right\} \quad\Longrightarrow\quad \mathcal{H}'_{\texttt{H}} \not\subset \mathcal{A}'_{\texttt{A}} \not\subset \Omega'_{\texttt{O}} \ . \tag{1.2.6}$$

Now, suppose $\mathcal{D}_{\texttt{H}}$, $\mathcal{D}_{\texttt{A}}$, and $\mathcal{D}_{\texttt{O}}$ are three non-intersecting subsets of $\mathbb{R}$ such that

$$\begin{aligned} \psi\in\mathcal{H}'_{\texttt{H}} \quad &\Longrightarrow \quad \psi:\mathcal{D}_{\texttt{H}}\to\mathbb{C} \\ \psi\in\mathcal{A}'_{\texttt{A}} \quad &\Longrightarrow \quad \psi:\mathcal{D}_{\texttt{A}}\to\mathbb{C} \\ \psi\in\Omega'_{\texttt{O}} \quad &\Longrightarrow \quad \psi:\mathcal{D}_{\texttt{O}}\to\mathbb{C} \ . \end{aligned} \tag{1.2.7}$$

A function is usually defined as a binary relation between two sets so it follows, for instance, that $\psi(x)=\sin(x)$ is the same function regardless of which $\mathcal{D}$ is its domain. However, if

$$\begin{aligned} \mathcal{H}'_{\texttt{H}} &\ni \psi_{\texttt{H}}:[0,2\pi]\to[-1,1] \\ \mathcal{A}'_{\texttt{A}} &\ni \psi_{\texttt{A}}:[4\pi,6\pi]\to[-1,1] \\ \Omega'_{\texttt{O}} &\ni \psi_{\texttt{O}}:[8\pi,10\pi]\to[-1,1] \ , \end{aligned} \tag{1.2.8}$$

then the different ψ_k are not exactly the same. This invokes a nuanced technical issue which we will revisit in Section 31 pertaining to a criticism of Scholze and Styx against Mochizuki's inter-universal Teichmüller theory (IUT.)[1] The definition of a function as a binary relation between two sets makes it easy to ignore the subtle distinction between a state space containing abstract $|\psi\rangle$ vectors and function spaces containing the $\psi(x)$ wavefunction representations. It is normal in physics to write $|\psi\rangle=\psi(x)$ meaning that the state is identically the wavefunction. Formally, it is not. To be very specific, or rigorous, one must ask if the definition of the function includes the identity of the two sets related by it. Regarding the matter of $\hat{M}^3$, it is not relevant whether the identity of a function depends on the identity of its domain. The nested structure of $\{\mathcal{H}',\mathcal{A}',\Omega'\}$ is such that $\psi\in\mathcal{H}'$ implies $\psi\in\mathcal{A}'$ and $\psi\in\Omega'$, and we will do physics in the way that ignores unnecessary mathematical nuance. We will drop the subscripts and call $\{\mathcal{H}',\mathcal{A}',\Omega'\}$ an RHS even though we have added an implicit labeling scheme such that the nested structure is broken by (1.2.4), in some sense.

MCM state spaces must have an associated manifold specified so we may know

[1] We will suggest that Mochizuki's Hodge theater is a rebranded MCM unit cell and that IUT is an attempted completion of the $\hat{M}^3$ theory.

which coordinates chart the domains of the states' wavefunction representations. For this purpose, we have introduced the ontological basis $\{\hat{e}_\mathcal{H}, \hat{e}_\mathcal{A}, \hat{e}_\Omega\}$ such that

$$\begin{aligned}
\psi \in \mathcal{H}' &\qquad\iff\qquad \big|\psi\big\rangle = \big|\psi\big\rangle\hat{e}_\mathcal{H} = \big|\psi;\hat{e}_\mathcal{H}\big\rangle = \psi(x^i) \\
\psi \in \mathcal{A}' &\qquad\iff\qquad \big|\psi\big\rangle = \big|\psi\big\rangle\hat{e}_\mathcal{A} = \big|\psi;\hat{e}_\mathcal{A}\big\rangle = \psi(x^i_-) \\
\psi \in \Omega' &\qquad\iff\qquad \big|\psi\big\rangle = \big|\psi\big\rangle\hat{e}_\Omega = \big|\psi;\hat{e}_\Omega\big\rangle = \psi(x^i_+) \quad .
\end{aligned} \tag{1.2.9}$$

We also suppose the existence of a fourth basis element $\hat{e}_\varnothing$ such that $|\psi\rangle\hat{e}_\varnothing = \psi(x^i_\varnothing)$ or $|\psi\rangle\hat{e}_\varnothing = \psi(\chi^a_\varnothing)$. (Refer to Figure 1 for placement of $\varnothing$ in the unit cell.) Now that we have developed the requisite objects, we may supplement the abstract notation of (1.2.2) and (1.2.3) with an ordinary operator algebra. Letting $\hat{M}^3 \equiv \hat{M}_3\hat{M}_2\hat{M}_1$, we have

$$\left.\begin{aligned}
\hat{M}_1\big|\psi;\hat{e}_{\mathcal{H}_1}\big\rangle &= c_1\big|\psi;\hat{e}_{\Omega_1}\big\rangle \\
\hat{M}_2\big|\psi;\hat{e}_{\Omega_1}\big\rangle &= c_2\big|\psi;\hat{e}_{\mathcal{A}_2}\big\rangle \\
\hat{M}_3\big|\psi;\hat{e}_{\mathcal{A}_2}\big\rangle &= c_3\big|\psi;\hat{e}_{\mathcal{H}_2}\big\rangle
\end{aligned}\right\} \qquad\Longrightarrow\qquad \hat{M}^3\big|\psi;\hat{e}_{\mathcal{H}_1}\big\rangle = c_3c_2c_1\big|\psi;\hat{e}_{\mathcal{H}_2}\big\rangle \quad . \tag{1.2.10}$$

$\hat{M}^3$ executes $\mathcal{H}_1 \to \mathcal{H}_2$ via the given intermediate steps. It operates on states in one unit cell and returns states in a time-advanced unit cell. Schrödinger evolution also occurs between $\mathcal{H}_1$ and $\mathcal{H}_2$. The intermediate steps of $\hat{M}^3$ are specified to add complexity to the usual theory in which the Schrödinger equation is integrated from t_1 to t_2. Assigning $t_1 \in \mathcal{H}_1$ and $t_2 \in \mathcal{H}_2$,[1] the intermediate steps provide a framework in which more can happen than what QM describes as monotonic diffusion followed by instantaneous collapse. The structure provided by the intermediate steps is pointed out so as to avoid an appearance of redundancy in what might otherwise be written as $\hat{M}^3 : \mathcal{H}_1 \to \mathcal{H}_2$ without a reference to the intermediate steps that should be useful for applications towards modified Schrödinger evolution. Further inquiry is required to determine an analytical statement of this new theoretical structure.

In practice, the MCM cosmological lattice is infinite in extent. Each unit cell resides at a later chronological time than all leftward unit cells, and at a later chirological time. Each successive unit cell is said to be on a higher *level of aleph* (Section 1.6) [2, 48] than the unit cells at earlier chirological times. Levels of aleph are an abstract characteristic introduced to differentiate one unit cell from its neighbors. The subscripts on the $\{\hat{e}_\mu\}$ in (1.2.10), e.g.: $\hat{e}_{\Omega_1}$ and $\hat{e}_{\mathcal{A}_2}$, refer to branes on the first and second levels of aleph. (See Figure 1 for similar labeling on $\Sigma^\pm$.) Levels of aleph

[1] This notation means that the measurement associated with $\mathcal{H}_1$ happened at $x^0 = t_1$, t_1 was the observer's proper time in $\mathcal{H}_1$, and the same for t_2 and $\mathcal{H}_2$.

are labeled with integers so any $\mathcal{H}_k$ will have an infinite number of earlier and later $\{\mathcal{H}_j\}$.

In practice, it may be useful to consider cyclic $\hat{M}^3 : \mathcal{H} \to \Omega \to \mathcal{A} \to \mathcal{H}$ in place of the non-cyclic $\hat{M}^3 : \mathcal{H}_1 \to \Omega_1 \to \mathcal{A}_2 \to \mathcal{H}_2$. In other words, we might drop the subscripts to treat the problem as a small algebraic group.

1.2.1 A Program in Number Theory

$\hat{M}^3$ is formulated to describe the process by which a theory is tested with experiment. The operation is *psychological* because the chronological time interval between two unit cells depends on how long the observer waits to test his prediction. A requirement for regular periodicity in the overall lattice of all unit cells, or for the *self-similarity* of all unit cells, is fulfilled through a regularized chirological time interval between $\mathcal{H}_1$ and $\mathcal{H}_2$. The interval in the abstract coordinates will be proportional to the golden ratio Φ without regard for the duration of chronological time between successive measurements. We will say more about the golden ratio and our reasons for using it in Section 1.2.4 (see also [70, 71]).

The defining property of a set of basis vectors is the linear independence of the basis' elements. Usually, the elements are unit vectors. The particular basis $\{\hat{e}_\mathcal{H}, \hat{e}_\mathcal{A}, \hat{e}_\Omega, \hat{e}_\varnothing\}$ is called "ontological" due to the specification of certain non-unit magnitudes for its elements. By choosing the number-theoretically significant magnitudes $\{2, \pi, i, \Phi\}$ in some order for $\{\|\hat{e}_\mathcal{H}\|, \|\hat{e}_\Omega\|, \|\hat{e}_\mathcal{A}\|, \|\hat{e}_\varnothing\|\}$ we hope to generate certain properties of the natural world by these numbers' association with the structure of the unit cell. The present convention is

$$\begin{array}{cccc} \hat{e}_\mathcal{H} = \hat{\pi} & \hat{e}_\Omega = \hat{\Phi} & \hat{e}_\mathcal{A} = \hat{2} & \hat{e}_\varnothing = \hat{i} \\ |\hat{\pi}| = \pi & |\hat{\Phi}| = \Phi & |\hat{2}| = 2 & |\hat{i}| = 1 \\ \|\hat{\pi}\| = \pi & \|\hat{\Phi}\| = \Phi & \|\hat{2}\| = 2 & \|\hat{i}\| = i \end{array} \quad .^1 \tag{1.2.11}$$

Using a further convention such that the observer's reference frame at measurement A is normalized to the zeroth level of aleph, $\hat{M}^3$ will operate as

$$\left.\begin{aligned} \hat{M}_1\big|\psi;\hat{\pi}^0\big\rangle &= \pi\big|\psi;\hat{\Phi}^0\big\rangle \\ \hat{M}_2\big|\psi;\hat{\Phi}^0\big\rangle &= \Phi\big|\psi;\hat{2}^1\big\rangle \\ \hat{M}_3\big|\psi;\hat{2}^1\big\rangle &= 2\big|\psi;\hat{\pi}^1\big\rangle \end{aligned}\right\} \quad \Longrightarrow \quad \hat{M}^3\big|\psi;\hat{\pi}^0\big\rangle = 2\pi\Phi\big|\psi;\hat{\pi}^1\big\rangle \quad . \tag{1.2.12}$$

[1] In the previous conventions for the ontological basis vectors [1,3,30], $\mathcal{A}$ and $\varnothing$ were oppositely labeled with $\hat{i}$ and $\hat{2}$. The present convention is better suited to the MCM formula for the fine structure constant (Section 1.9).

$\hat{M}^3$ takes a state in one unit cell, or on one level of aleph, and puts it into the next one. This operator algebra is presented as an ansatz pending development of analytical representations for $\hat{M}^3$ and $|\psi;\hat{e}^k_\mu\rangle$. The former iterator subscript of (1.2.10) has been refashioned as an algebraically meaningful integer exponent k. $\hat{\pi}^0=\hat{1}$ allows us to use ordinary QM states as MCM states in $\mathcal{H}_0$.[1] For now, we will assume the $2\pi\Phi$ scalar coefficient[2] and proceed to examine the ontological basis.

To detail the basis' functioning, we will use the example

$$\mathbf{x} = x\hat{e}_x \quad . \tag{1.2.13}$$

It is understood that we may ignore the unit vector in the x direction to use notation such that $\mathbf{x}$ is a vector with magnitude x in the implicit direction. Similarly, we will recover ordinary QM state vectors in $\mathcal{H}$ by ignoring $\hat{\pi}$:

$$\big|\psi;\hat{\pi}\big\rangle = \big|\psi\big\rangle\hat{\pi} = \psi(x^i) \quad . \tag{1.2.14}$$

If the normalization convention includes the magnitude of $\hat{\pi}$, i.e.:

$$\big\langle\psi;\hat{\pi}\big|\psi;\hat{\pi}\big\rangle = |\hat{\pi}|\big\langle\psi\big|\psi\big\rangle|\hat{\pi}| = 1 \quad , \tag{1.2.15}$$

then the convention of (1.2.14) induces a notion of relative scale between branes. For instance, (1.2.14) would be written

$$\big|\psi;\hat{\pi}^0\big\rangle = \big|\psi\big\rangle\hat{\pi}^0 = \psi(x^i)|\hat{\pi}^0| \quad , \tag{1.2.16}$$

while ignoring a non-unit basis vector will alter a state's magnitude:

$$\big|\psi;\hat{\pi}^1\big\rangle = \big|\psi\big\rangle\hat{\pi}^1 = \frac{1}{\pi}\psi(x^i)|\hat{\pi}^1| \quad , \quad \text{and} \qquad \big|\psi;\hat{\pi}^k\big\rangle = \big|\psi\big\rangle\hat{\pi}^k = \frac{1}{\pi^k}\psi(x^i)|\hat{\pi}^k| \quad . \tag{1.2.17}$$

This concept of relative scale will be used extensively in later sections.

A further property of the hat notation is demonstrated with the redundant expression

$$\mathbf{a} = a\,\hat{e}_x\,|\hat{e}_y||\hat{e}_z| \quad , \quad \text{where} \qquad |\hat{e}_i| = 1 \quad . \tag{1.2.18}$$

If one wants to know what $\mathbf{a}$ looks like when it points in in the y direction, call it $\mathbf{a}'$,

[1] $\hat{\pi}^0=\hat{1}$ may suggest that $\hat{\Phi}^0=\hat{1}$ as well. To avoid any possible association of $\hat{\Phi}^0$ with the identity operator, future inquiry might study the case where the Ω-brane following $\mathcal{H}_0$ is already on the higher level of aleph. The current labeling scheme is such that $\mathcal{H}$ and its adjacent $\mathcal{A}$- and Ω-branes are on the same level of aleph. The level is said to increase at $\varnothing$, as in Figure 1. However, an alternative convention in which the level of aleph increases at $\mathcal{H}$ must be considered as well. In that convention, all chirologically future-directed branes beyond $\mathcal{H}_0$ would be labeled by $k>1$ on ontological basis vectors with non-unit magnitudes.

[2] This scalar differs from the $i\pi\Phi$ and $i\pi\Phi^2$ constants which have appeared in previous work due mainly to the reassignments of $\hat{\imath}$ and $\hat{2}$.

one must rearrange the absolute value bars. For some operator $\hat{\mathsf{O}}_{x\to y}$, we have

$$\hat{\mathsf{O}}_{x\to y}\mathbf{a} = \hat{\mathsf{O}}_{x\to y}\big(a\,\hat{e}_x|\hat{e}_y||\hat{e}_z|\big) = a\,|\hat{e}_x|\,\hat{e}_y\,|\hat{e}_z| = \mathbf{a}' \quad . \tag{1.2.19}$$

Usually, $\hat{\mathsf{O}}_{x\to y}$ would be a $\pi/2$ rotation operation about the z-axis but here we wish to emphasize an algebraic picture over a geometric one. In the desired algebraic picture, we have an implicit similitude to the three steps in (1.2.12):

$$\hat{M}_1 \sim \hat{\mathsf{O}}_{\mathcal{H}\to\Omega} \quad , \quad \hat{M}_2 \sim \hat{\mathsf{O}}_{\Omega\to\mathcal{A}} \quad , \quad \text{and} \qquad \hat{M}_3 \sim \hat{\mathsf{O}}_{\mathcal{A}\to\mathcal{H}} \quad . \tag{1.2.20}$$

The laws of linear algebra suggest that we may execute any $\hat{\mathsf{O}}_{\mu\to\nu}$ simply by moving the hat around. The matter is slightly complicated in the unit cell by the non-unit magnitudes of the ontological basis vectors but the procedure will follow (1.2.19). To preserve the unit magnitude of the identity in the following, we will replace the $|\hat{e}_i|$ of (1.2.19) with $\|\hat{e}_\mu\|/\|\hat{e}_\mu\|$. Considering $|\hat{i}|=1$ and $\|\hat{i}\|=i$, the norm rather than the absolute value is used to write, for example,

$$\begin{aligned} \hat{\mathsf{O}}_{\mathcal{H}\to\mathcal{A}}\big|\psi;\hat{\pi}\big\rangle &= \hat{\mathsf{O}}_{\mathcal{H}\to\mathcal{A}}\big|\psi\big\rangle\hat{\pi}\,\frac{\|\hat{2}\|}{\|\hat{2}\|}\frac{\|\hat{\Phi}\|}{\|\hat{\Phi}\|}\frac{\|\hat{i}\|}{\|\hat{i}\|} \\ &= \big|\psi\big\rangle\|\hat{\pi}\|\frac{\hat{2}}{\|\hat{2}\|}\frac{\|\hat{\Phi}\|}{\|\hat{\Phi}\|}\frac{\|\hat{i}\|}{\|\hat{i}\|} \\ &= \frac{\pi}{2}\,\big|\psi;\hat{2}\big\rangle \quad . \end{aligned} \tag{1.2.21}$$

More concisely, one inserts the relevant identity and moves the hat:

$$\begin{aligned} \hat{\mathsf{O}}_{\mathcal{H}\to\mathcal{A}}\big|\psi;\hat{\pi}\big\rangle &= \hat{\mathsf{O}}_{\mathcal{H}\to\mathcal{A}}\left(\mathbb{1}\big|\psi\big\rangle\hat{\pi}\right) \\ &= \hat{\mathsf{O}}_{\mathcal{H}\to\mathcal{A}}\left(\frac{2}{2}\,\big|\psi\big\rangle\hat{\pi}\right) \\ &= \frac{\pi}{2}\,\big|\psi;\hat{2}\big\rangle \quad . \end{aligned} \tag{1.2.22}$$

This protocol for moving hats will be integral to the MCM prescription for quantum gravity in Section 1.10.1.

1.2.2 An Example in Atomic Physics

The commonality of 2, π, and i in quantum theory's analytical expressions motivates their placement in the ontological basis. For example, the wavefunction of a hydrogenic electron ψ_{nlm} is such that

$$\psi_{100} = \frac{1}{\sqrt{4\pi}}\frac{2}{a_0^{3/2}}\,e^{-r/a_0} \quad , \quad \text{and} \qquad \psi_{211} = \frac{1}{\sqrt{64\pi}}\frac{1}{a_0^{3/2}}\,e^{-r/2a_0}\sin(\theta)e^{i\phi} \quad . \qquad (1.2.23)$$

The numbers 2, π, and i are analytically integral in such expressions. On the other hand, the absent number Φ is associated with the χ^4 direction that is absent from the usual framework for QM. An appeal to the arena of QM as the zeroth level of aleph shows that $\Phi^0=1$ is already present in ψ_{nlm}, and every other conceivable wavefunction. The MCM seeks to *modify* the usual arena for quantum theory by embedding it in a fifth dimension. States enter the new MCM arena along $\hat{\Phi}$ pointing out of $\mathcal{H}$ in the χ^4_+ direction toward Ω. As Ω will be located at $\chi^4_+=\Phi$, we may expect that factors of Φ will accrue upon successive applications of $\hat{M}^3$. This is already codified into the $2\pi\Phi$ constant given by $\hat{M}^3|\psi;\hat{\pi}^0\rangle=2\pi\Phi|\psi;\hat{\pi}^1\rangle$. Such factors of Φ^k will be as integral to the analytical representations of wavefunctions in non-$\mathcal{H}_0$ branes as are 2, π, and i in $\mathcal{H}_0$.[1]

In the convention such that $|\psi;\hat{\pi}\rangle=|\psi\rangle\hat{\pi}=\psi(x^i)$, we have hydrogenic states

$$\big|n,l,m;\hat{\pi}\big\rangle = \big|n,l,m\big\rangle\hat{\pi} = \psi_{nlm}(r,\theta,\phi) \quad , \qquad (1.2.24)$$

where $\{r,\theta,\phi\}$ are the spherical polar representation of $x^i\in\mathcal{H}$. Using ψ_{100} as an example, $\hat{M}^3$ operates as

$$\hat{M}^3\big|1,0,0;\hat{\pi}^0\big\rangle = 2\pi\Phi\big|1,0,0;\hat{\pi}^1\big\rangle = \sqrt{4\pi}\,\frac{\Phi}{a_0^{3/2}}\,e^{-r/a_0} \quad . \qquad (1.2.25)$$

ψ_{nlm} is not time-dependent so the wavefunction must be the same across any number of successive measurements. As a result, (1.2.25) is mathematically trivial. On the other hand, the theory of quantum states in Hilbert space is such that any two states which differ by a constant are the same state. Therefore, (1.2.25) satisfies an important physical constraint: the stationary state remains stationary.

Regarding time-dependent states, it is expected that the ∂_0 and/or ∂_4 time derivatives are the generators of $\hat{M}^3$. The case of $\hat{M}^3$ acting on time-dependent states must be more complicated than the example of ψ_{nlm} in which all such derivatives vanish. In general, the structure of the unit cell is such that measurement B in $\mathcal{H}_1$ occurs at a later chronological time than measurement A in $\mathcal{H}_0$. Consequently, it is required that we start with $|\psi,t_0,\hat{\pi}^0\rangle$ and end with $|\psi,t_1;\hat{\pi}^1\rangle$ for some $t_1>t_0$. At minimum, $\hat{M}^3$ must be complemented with Schrödinger evolution. More likely, $\hat{M}^3$ has its own

[1] Later, we will suggest that the exponent on $\hat{\Phi}$ should describe differences in the level of aleph so $\hat{\Phi}^{\Delta k}$ should vanish for physics confined to $\mathcal{H}_0$.

unique time evolution equation which contains the Schrödinger equation as the limit of vanishing chirological derivatives.

1.2.3 The Proton Radius in Muonic Hydrogen

An unsolved anomaly in modern physics is that the proton radius measured in muonic hydrogen is different than the proton radius measured in electronic hydrogen [64]. Such a result might be explained in principle as a corollary of (1.2.25) because MCM muons live in a different state space than MCM electrons (Section 0.3). In the way that one obtains an arbitrary momentum state by applying a boost to a $k=0$ state, one would obtain a muon state from an electron by applying some $\hat{\mathsf{O}}_{\hat{e}_\mu\to\hat{e}_\nu}$ in the sense of (1.2.19). This operation would have its own non-unit magnitude scalar constant associated with it because $2\pi\Phi$ is uniquely associated with $\hat{M}^3\sim\hat{\mathsf{O}}_{\mathcal{H}_k\to\mathcal{H}_{k+1}}$. By some more complicated mechanism, that constant might manifest as an observably different proton radius in the muon-nucleon bound state. Given the normalization convention in (1.2.15) and a proton radius operator $\hat{r}_p$, one would obtain various matrix elements

$$\left(\hat{r}_p\right)_{\mu\nu} = \left\langle\psi;\hat{e}_\mu\middle|\hat{r}_p\middle|\psi;\hat{e}_\nu\right\rangle \quad , \tag{1.2.26}$$

for ψ in various branes. For $\mu=\nu$, these matrix elements reduce to the expectation value $\langle\hat{r}_p\rangle$.

1.2.4 The $\hat{\varphi}$ Object and $\mathbb{C}^*$

The piecewise assembly of the unit cell in Figure 4a makes $\chi^4_\pm$ appear to be linearly dependent. However, these are two linearly independent degrees of freedom. We will take $\hat{\varphi}$ to point in the χ^4_- direction while $\hat{\Phi}$ points in the direction of χ^4_+. The right angle in Figure 4b depicts a unit cell assembled from subdomains of two orthogonal, unbounded intervals of $\chi^4_\pm$.

We have proposed a convention in which $\mathcal{A}$ and Ω are located at $\chi^4_-=-\varphi$ and $\chi^4_+=\Phi$ relative to $\mathcal{H}$ at $\lim\chi^4_\pm\to 0$. Assuming that $\mathcal{H}$ is spanned by one unit of x^0, the $\Phi\times 1$ and $1\times\varphi$ dimensions of the $\chi^4_- x^0$ and $\chi^4_+ x^0$ boxes makes each an identical golden rectangle. By the well known properties of the golden ratio, $\Phi^k\times\Phi^{k-1}$ is the only aspect ratio that will allow an infinite tiling succession of different-sized unit cells, each in the same proportion.[1] The infinite succession of unit cells is called **the cosmological lattice**. A unit scale such that each unit cell is the same size as the others generates a constant proportion of self-similarity but the golden ratio uniquely

[1] Physical conventions for increasing wavenumber along a golden spiral progression of unit cells were developed in [70].

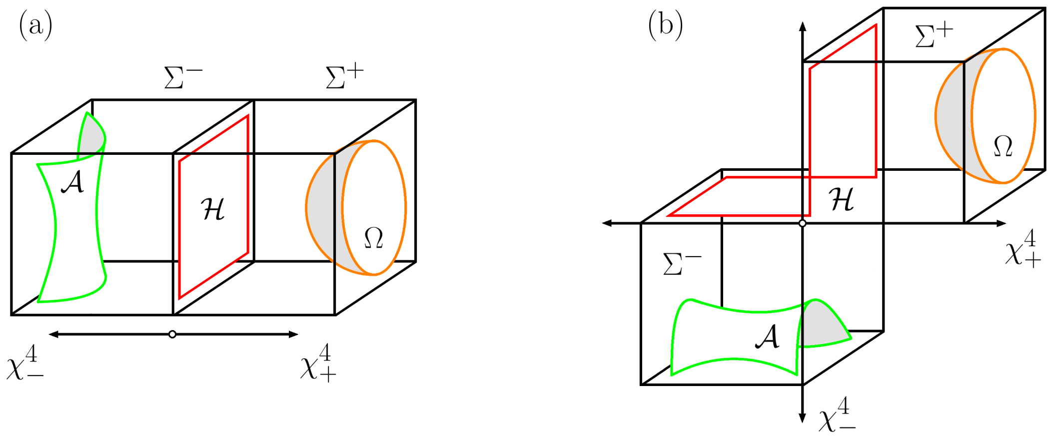

Figure 4: Figure (b) shows an arrangement in which negative-definite $\chi^4_- \in \Sigma^-$ and positive-definite $\chi^4_+ \in \Sigma^+$ might be assembled from two orthogonal, unbounded intervals of $\chi^4_\pm$. Compared to (a), (b) better emphasizes the linear independence of χ^4_+ and χ^4_-.

allows a non-unit tiling proportion:

$$\Phi = \frac{b}{a} = \frac{a+b}{b} \quad . \tag{1.2.27}$$

Non-constant scale across successive unit cells is considered desirable for the generation of an arrow of time, and for other results such as the MCM mechanism for dark energy (Section 7). In a unit scaling, we might appeal to the cosmological constant $\Lambda \propto R$ to say that $\mathcal{A}$ has lower energy than Ω and that, therefore, the chirological arrow of time should point to the left from $\mathcal{H}$. However, the energy would have to increase again from Σ^- passing into Σ^+ on the round trip back to $\mathcal{H}$ (barring some more nuanced convention for dynamics at $\varnothing$.) The presently assumed convention is that states go into Ω before $\mathcal{A}$. To support that condition, we will implement a non-unit scale such that $\hat{M}^3$ preferentially moves states toward the right in the cosmological lattice. Although there does not exist an accepted energy landscape setting the chronological arrow of time, increasing volume in future-directed unit cells may set a chirological arrow of time pointing toward the right based on the thermodynamic tendency of energy densities to decrease.[1] If the forward scale should be smaller, we might invoke gravitational collapse into a singularity at $\varnothing$ to favor a rightward arrow. The main principle is that any scale other than unit scaling can be used to support

[1]Early steady state models in cosmology supposed a constant generation of new matter-energy to maintain constant density under Hubble expansion [72, 73]. Non-unitary MCM time evolution discussed in Section 1.2.5 may serve a similar purpose.

an arrow of time. Furthermore, non-unit scale will be required to restore normalized probability amplitudes after non-unitary evolution under $\hat{M}^3$ (Section 1.2.5). By synergy, one would hope to connect these two cases for non-unit scale. As it relates to the present section, $\hat{\Phi}$ points in the direction of increasing scale and $\hat{\varphi}$ points in the direction of decreasing scale.

The $\{x^0, x^i, \chi^4_\pm\}$ orthogonal coordinate triads are distinguished as right- or left-handed when $\chi^4_\pm$ are associated with oppositely directed chirological time by $\hat{\Phi}$ and $\hat{\varphi}$. $\{x^0, x^i, \chi^4_+\}$ is right-handed and $\{x^0, x^i, \chi^4_-\}$ is left-handed. These orthogonal triads are said to span $\mathbb{C}^*_\pm$ in $\Sigma^\pm$ respectively. The unit cell is extended from $\mathbb{C}$ in the transverse direction by $\hat{\Phi}$ pointing to the right, and by $\hat{\varphi}$ pointing to the left or down. $\mathbb{C}$ and its transverse continuations are called $\mathbb{C}^*_\pm$. To briefly clarify $\mathbb{C}^*_\pm$ without fully formalizing it, and to indicate an avenue for productive future inquiry into distinctness between $\hat{\varphi}$ and $\hat{\Phi}$, the complex plane $\mathbb{C}$ spanned by $\hat{1}$ and $\hat{\imath}$ is extended in the $\hat{\Phi}$ transverse direction and/or the $\hat{\varphi}$ transverse direction. Using identities $\hat{x} = \hat{x}^i$ and $ic\hat{t} = \hat{x}^0$,[1] we may associate $\mathcal{H}$ with $\mathbb{C}$. Suppressing two spatial dimensions, $\hat{x}$ and $\hat{t}$ point in the $\hat{1}$ and $\hat{\imath}$ directions respectively. This convention for imaginary t is required to obtain the requisite minus sign in the differential element of flat spacetime interval:[2]

$$\begin{aligned} ds^2 &= \big(dx^0\big)^2 + \big(dx^1\big)^2 + \big(dx^2\big)^2 + \big(dx^3\big)^2 \\ &= -c^2\, dt^2 + dx^2 + dy^2 + dz^2 \quad . \end{aligned} \tag{1.2.28}$$

The quadratic relationship $(dx^0)^2 = -c^2 dt^2$ implies a factor of i in the linear relationship. Compared to the convention where the metric is assumed as $g_{\mu\nu} = \text{diag}(-c^2, 1, 1, 1)$ a priori, the convention for $x^0 = ict$ is superior for a number of reasons including its facilitation of the present association between $\mathcal{H}$ and $\mathbb{C}$.

The extended complex conjugation algebra for $\mathbb{C}^*_\pm$ is

$$\left.\begin{aligned} \hat{\varphi}^* &= \hat{\Phi} \\ \hat{\Phi}^* &= -i\hat{\varphi} \end{aligned}\right\} \quad \Longrightarrow \quad \big(\hat{\varphi}^*\big)^* = -i\hat{\varphi} \neq \hat{\varphi} \quad .^3 \tag{1.2.29}$$

This is intended to introduce a quality of irreversibility into progression across the unit cell [30]. Referring to Figure 4a, the basis vectors pointing to the left and right of $\mathcal{H}$ are not merely sign conjugates as are $\{\hat{1}, -\hat{1}\}$ and $\{\hat{\imath}, -\hat{\imath}\}$ pointing in the

[1] This notation for imaginary t relative to real x^0 may be found in Appendix A3–2 of [74], for example.

[2] The relationship between negative metric signature and imaginary dimension, or imaginary dimensional transposing parameter, is treated again in Section 10.

[3] Φ is a real number so the meaning of the $*$ operator in $\mathbb{C}^*$ must not be confused with its context in $\mathbb{C}$ where Φ^* is equal to Φ.

directions that span $\mathcal{H}$. Due to this assumed conjugation algebra, $\hat{M}^3$ and $(\hat{M}^3)^\dagger$ are not expected to raise and lower the level of aleph as the Dirac ladder operators $\hat{a}$ and $\hat{a}^\dagger$ raise and lower the principal quantum number for simple harmonic oscillator states. Figure 4b makes it easy to envision $(\hat{M}^3)^\dagger$ as sending a state in $\mathcal{H}$ into an upward instance of Σ^- other than the downward one from which it came.

At first glance, χ^4_- must be imaginary relative to real χ^4_+ because $\chi^4_\pm$ are oppositely timelike and spacelike in the KK metric. For $A^\mu_\pm = 0$, we have

$$g^\pm_{AB} = \begin{pmatrix} g^\pm_{\alpha\beta} & 0 \\ 0 & \chi^4_\pm \end{pmatrix} = \begin{pmatrix} -c^2 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & \pm|\chi^4_\pm| \end{pmatrix} \quad , \tag{1.2.30}$$

where $\chi^4_- = -|\chi^4_-|$ because χ^4_- is negative-definite in Σ^-. This metric implies

$$ds_+ \propto \sqrt{\chi^4_+}\, d\chi^4_+ \quad , \qquad \text{and} \qquad ds_- \propto i\sqrt{|\chi^4_-|}\, d\chi^4_- \quad .^1 \tag{1.2.31}$$

The minus sign on $g^\pm_{00}$ requires that distance in the x^0 direction is imaginary (timelike) relative to real spatial distance in the x^i directions. Likewise, the minus sign on g^-_{44} requires that χ^4_- is imaginary relative to χ^4_+. To preserve the timelike character of χ^4_+, we might alternate the phase convention in successive unit cells or associate spacelike χ^4_+ with the imaginary time of statistical mechanics. Overall, the changing metric signature between $\Sigma^\pm$ represents a hard problem in the issue of the forward connection of Σ^+ to Σ^- but the issue is well contextualized in the assignment of the $\hat{i}$ ontological basis vector to $\varnothing$. An extra factor of i may be what is needed to resolve the topological mismatch between the number of spacelike and timelike dimensions in $\Sigma^\pm$. Furthermore, Figure 4 suggests that we might define $i\chi^4_\pm$ as two mutually orthogonal directions pointing out of the page such that $\hat{M}^3$ weaves a path along $\chi^4_\pm \in \mathbb{C}$ where no metric signature discrepancies are present. To accomplish this, we would rely on the free sign in the Lorentzian metric signature $\{\mp\pm\pm\pm\}$ to alternately assign the factor of i to the real and imaginary parts of $\chi^4_\pm$ in successive unit cells. In some sense, we might use the $i = e^{i\pi/2}$ identity to associate the $\hat{i}$ ontological specifier for $\varnothing$ with a $\pi/2$ rotation away from the direction of metric discrepancy.

[1] Rather than the $\phi^2_\pm = \chi^4_\pm$ convention shown in (1.2.30), if we require that an alternative convention for $\phi_\pm = \chi^4_\pm$ preserves the $\{-+++-\}$ signature in Σ^-, which is required if g^-_{44} is the negative Ricci scalar of AdS$_4$, then we obtain the complex phase in (1.2.31) more naturally without the square roots.

In Section 1.7.3, we will associate the region of metric discrepancy with an energetically forbidden region in which the potential energy is higher than the total energy. If the metric discrepancy is associated with real $\chi_+^4 \in \Sigma^+$ followed by imaginary $\chi_-^4 \in \Sigma^-$, the energy landscape will be such that MCM plane wave solutions are preferentially steered onto the branch of $\chi_-^4 \in \mathbb{C}$ which is real, thus avoiding the metric discrepancy.

Finally, we have presented $\hat{\Phi}$ as an ontological basis vector and as a geometric basis vector pointing in the direction of χ_+^4. We will go on to develop a picture of the ontological basis vectors $\{\hat{2}, \hat{\pi}, \hat{\imath}, \hat{\Phi}\}$ as lattice vectors anchored in each labeled brane. These vectors will span an ontological lattice in the usual sense of crystallography.

1.2.5 The Non-Unitary Property of $\hat{M}^3$

In quantum mechanics, an operator $\hat{\mathsf{U}}$ is unitary if $\hat{\mathsf{U}}^\dagger \hat{\mathsf{U}} = \mathbb{1}$. It is unitary if the inverse is the conjugate transpose, also called the Hermitian conjugate or the adjoint. For a time-independent Hamiltonian, the unitary time evolution operator which satisfies Schrödinger's equation is

$$\hat{\mathcal{U}}(t_1, t_0) = \exp\left\{ \frac{-i\hat{H}\big(t_1 - t_0\big)}{\hbar} \right\} \quad , \quad \text{such that} \qquad \hat{\mathcal{U}}(t_1, t_0)\big|\psi, t_0\big\rangle = \big|\psi, t_1\big\rangle \quad . \tag{1.2.32}$$

The main application of the unitary property in quantum physics is that the probability interpretation of the wavefunction is preserved by unitary operations. Given

$$\big\langle \psi, t_0 \big| \psi, t_0 \big\rangle = \int_{-\infty}^{\infty} dx\, \psi^*(x, t_0)\psi(x, t_0) = 1 \quad , \tag{1.2.33}$$

meaning that the probability of observing ψ somewhere in the universe is 100% at time t_0, the unitary evolution operator is such that

$$\big\langle \psi, t_0 \big| \hat{\mathcal{U}}^\dagger(t_1, t_0)\, \hat{\mathcal{U}}(t_1, t_0) \big| \psi, t_0 \big\rangle = \big\langle \psi, t_1 \big| \psi, t_1 \big\rangle = 1 \quad . \tag{1.2.34}$$

After undergoing unitary evolution to an arbitrary time t_1, the probability of finding ψ somewhere in the universe is still 100%. The probability obtained in (1.2.33) was multiplied by a factor of *unity* in (1.2.34).

It was emphasized in the development of the MCM that $\hat{M}^3$ is *not* a unitary operator. The inverse of $\hat{M}^3$ is not its conjugate transpose and it should not preserve the probability interpretation without supplemental considerations. If the inverse of

$\hat{M}^3$ exists,

$$\big(\hat{M}^3\big)^{-1}\hat{M}^3=\hat{M}^3\big(\hat{M}^3\big)^{-1}=\mathbb{1}\qquad\Longrightarrow\qquad\big(\hat{M}^3\big)^{-1}\big|\psi;\hat{\pi}^k\big\rangle=\frac{1}{2\pi\Phi}\,\big|\psi;\hat{\pi}^{k-1}\big\rangle\quad.\tag{1.2.35}$$

The rules of matrix algebra are such that

$$\Big(\hat{M}^3\big|\psi;\hat{\pi}\big\rangle\Big)^\dagger=\big\langle\psi;\hat{\pi}^0\big|\big(\hat{M}^3\big)^\dagger=\big(2\pi\Phi\big)^*\big\langle\psi;\hat{\pi}^1\big|\quad.\tag{1.2.36}$$

The latter result may be combined with $\hat{M}^3$ operating to the right to show that the inverse is not the conjugate transpose:

$$\big\langle\psi;\hat{\pi}^0\big|\big(\hat{M}^3\big)^\dagger\hat{M}^3\big|\psi;\hat{\pi}^0\big\rangle=\big(2\pi\Phi\big)^*2\pi\Phi\qquad\Longrightarrow\qquad\big(\hat{M}^3\big)^\dagger\neq\big(\hat{M}^3\big)^{-1}\quad.\tag{1.2.37}$$

$\hat{M}^3$ is not a unitary operator.

Now we will suggest a context in which the non-unitary property of $\hat{M}^3$ will define unique MCM physics. Recalling

$$\begin{aligned}\hat{M}_1\big|\psi;\hat{\pi}^0\big\rangle&=\pi\big|\psi;\hat{\Phi}^0\big\rangle\\\hat{M}_2\big|\psi;\hat{\Phi}^0\big\rangle&=\Phi\big|\psi;\hat{2}^1\big\rangle\\\hat{M}_3\big|\psi;\hat{2}^1\big\rangle&=2\big|\psi;\hat{\pi}^1\big\rangle\quad,\end{aligned}\tag{1.2.38}$$

consider $\hat{M}_3\hat{M}_2\hat{M}_1=\hat{\boldsymbol{\pi}}\hat{\mathbf{2}}\hat{\boldsymbol{\Phi}}$ where $\hat{\boldsymbol{\Phi}}$ obeys

$$\left.\begin{aligned}\hat{\boldsymbol{\varphi}}^\dagger&=\hat{\boldsymbol{\Phi}}\\\hat{\boldsymbol{\Phi}}^\dagger&=-i\hat{\boldsymbol{\varphi}}\end{aligned}\right\}\qquad\Longrightarrow\qquad\hat{\boldsymbol{\varphi}}^{\dagger\dagger}=-i\hat{\boldsymbol{\varphi}}\neq\hat{\boldsymbol{\varphi}}\quad.\tag{1.2.39}$$

The general meaning of $\hat{\boldsymbol{\varphi}}$ is as in the previous section. It indicates the χ^4_- direction rather than the $-\chi^4_+$ direction. The bold operators are cast as

$$\hat{\boldsymbol{\Phi}}\sim\hat{\mathsf{O}}_{\mathcal{H}\to\Omega}\quad,\quad\hat{\mathbf{2}}\sim\hat{\mathsf{O}}_{\Omega\to\mathcal{A}}\quad,\quad\text{and}\qquad\hat{\boldsymbol{\pi}}\sim\hat{\mathsf{O}}_{\mathcal{A}\to\mathcal{H}}\quad.\tag{1.2.40}$$

Hermitian conjugation yields

$$\Big(\hat{\boldsymbol{\pi}}\,\hat{\mathbf{2}}\,\hat{\boldsymbol{\Phi}}\big|\psi;\hat{\pi}^k\big\rangle\Big)^\dagger=-i\,\big\langle\psi;\hat{\pi}^k\big|\hat{\boldsymbol{\varphi}}\,\hat{\mathbf{2}}^\dagger\,\hat{\boldsymbol{\pi}}^\dagger\quad.\tag{1.2.41}$$

This expression is intended to say that $\hat{\mathbf{2}}^\dagger$ and $\hat{\boldsymbol{\pi}}^\dagger$ will send states back the way they came through the cosmological lattice but $\hat{\boldsymbol{\varphi}}$ does not reverse $\hat{\boldsymbol{\Phi}}$. We may imagine that $\hat{\boldsymbol{\varphi}}$ sends the $\langle\psi;\hat{\pi}^k|$ bra up the χ^4_- number line (Figure 4) rather than back down in the direction from which it came. Therefore, one would write

$$\langle\psi;\hat{\pi}^k\big|\big(\hat{M}^3\big)^\dagger\hat{M}^3\big|\psi;\hat{\pi}^k\rangle = \langle\psi;\hat{\pi}^k\big|\big(\hat{\boldsymbol{\varphi}}\,\hat{\mathbf{2}}^\dagger\,\hat{\boldsymbol{\pi}}^\dagger\big)\hat{\boldsymbol{\pi}}\,\hat{\mathbf{2}}\,\hat{\boldsymbol{\Phi}}\big|\psi;\hat{\pi}^k\rangle = c\,\langle\psi;\hat{\pi}^{k''}\big|\psi;\hat{\pi}^{k'}\rangle \quad (1.2.42)$$

The k' and k'' notation at the right exposes what may be a shortcoming of the convention to assign levels of aleph to entire unit cells rather than individual branes. A single integer k is inadequate for labeling the branes which are off the beaten path of $\hat{M}^3$ (such as those indicated by $\hat{\boldsymbol{\varphi}}$). A more formal statement might include levels of aleph (quantum numbers) for all of the ontological basis vectors so that branes are labeled by sequences of integers. This more complicated MCM lattice structure is intuitive in Figure 4b but it is not needed in the usual representation of the unit cell.

$(\hat{M}^3)^\dagger$ has no ordinary use because the process of observation and measurement is constrained by a psychological arrow of time. A theory of making observations in reverse time order could never be tested, seemingly. However, the full analysis of a theory includes all possible operations and manipulations, such as time reversal operations which would come in chronological and chirological varieties. $\hat{\varphi}$ is introduced to make the chirological time reversal operator more than a trivial variation on the chronological one. It is considered desirable for physics that $\hat{M}^3$ and $(\hat{M}^3)^\dagger$ should have the sort of behavior inherent to the conjugation algebra for $\mathbb{C}^*$ because it represents a physical condition of time irreversibility. All possibilities for such functioning are predicated on the non-unitary property of $\hat{M}^3$. If $\hat{M}^3$ was unitary, call it $\hat{\mathsf{M}}^3$, then

$$\langle\psi;\hat{\pi}^k\big|\big(\hat{\mathsf{M}}^3\big)^\dagger\hat{\mathsf{M}}^3\big|\psi;\hat{\pi}^k\rangle = \langle\psi;\hat{\pi}^k\big|\mathbb{1}\big|\psi;\hat{\pi}^k\rangle = 1 \quad , \quad (1.2.43)$$

and there would be no possibility for more complicated behaviors. Thus, the non-unitary property is introduced in anticipation of further applications.

1.2.6 The Hierarchy Problem

The hierarchy problem asks about the origin of very large and very small numbers in physics. As an example, it asks why the weak force is more than 20 orders of magnitude stronger than gravitation. It is hoped that non-unitary chirological evolutions wherein effects such as tunneling and/or interference across various levels of aleph will motivate such disparate numerical scales. Very small numbers would pertain to lower levels of aleph $\sim(2\pi\Phi)^{-k}$ and large numbers would pertain to higher levels $\sim(2\pi\Phi)^k$. Such effects were previously invoked to compute the 10^{-4}m scale for new MCM physics (Section 15). In other sections, we will develop a case for infinite relative scale beyond the present irrational scale factor $2\pi\Phi$. If successive levels of aleph are associated with infinite relative scale, one might obtain appropriate hierarchical structures as the limits of uncertainty relationships where finite scale becomes indistinguishable from infinite scale.

1.2.7 Numerical Results

Responding to an observation that the ontological basis is chosen as a wild guess, it is pointed out that no less than three important dimensionless constants fall out of the choice without much complexity added in the path of computation.

- The fine structure constant α_{MCM} can be generated with the ontological numbers. α_{MCM} is treated in Section 1.9 where a ∼0.4% discrepancy with the accepted experimental value α_{QED} is discussed.
- The dimensionless constant 8π from Einstein's equation appears in a natural way as well (Section 1.10).
- The classical EM coupling constant $(4\pi)^{-1}$ appears in what is called **the ontological resolution of the identity**:

  $$\mathbb{1} \equiv \hat{1} = \frac{1}{4\pi}\,\hat{\pi} + \frac{\varphi}{4}\,\hat{\Phi} + \frac{1}{8}\,\hat{2} - \frac{i}{4}\,\hat{\imath} \quad . \tag{1.2.44}$$

  It is hoped that the ontological resolution of the identity will function as a scaffold on which to unify the four fundamental forces, or possibly the strong, weak, and EM forces with a hypothetical fifth force since gravitation is geometry, not force.

1.3 Tensor States

It was stated in [3] that MCM states specified with the ontological basis are tensor states. Proof that such states satisfy the tensor transformation law has not appeared previously. In this section, we will deviate from this book's theme of open problems to present a complete result: demonstration of tensor transformations for MCM states.

Wavefunctions satisfy the axioms of a vector space as follows.

- The vacuum state $|0\rangle$ is the zero vector $\vec{0}$.
- The sum (superposition) of two state vectors is another state vector.
- The (inner) product of two states is a non-state scalar.
- For a scalar c and a state $|\psi\rangle$, the product $c|\psi\rangle$ is still a state vector.

If there exist axioms of a tensor space, they are not so well known as the axioms of a vector space. To show that something is a tensor, one demonstrates the tensor transformation law which contains vector transformations as its simplest non-trivial case. However, it is not immediately intuitive that QM states satisfy the vector transformation law in the usual sense of coordinate transformations because the geometric picture of coordinates in state space plays little to no role in the ordinary practice of QM. Therefore, the structural framework for such a demonstration may enhance one's understanding of the theory. In this section, we will illuminate a little remarked upon feature of state spaces: they are coordinate spaces exactly like $\mathbb{R}^n$. Then we will make proofs of vector and tensor transformations for QM and MCM states respectively.

1.3.1 The Coordinates of State Space

To the extent that $\mathbb{R}^3$ is spanned by $\{\hat{x},\hat{y},\hat{z}\}$, an N-dimensional quantum state space is $\mathbb{R}^N$ spanned by $\{\hat{e}_1,\hat{e}_2,...,\hat{e}_N\}=\{|\psi_1\rangle,|\psi_2\rangle,...,|\psi_N\rangle\}$ where $\{|\psi_k\rangle\}$ is some orthonormal basis. The $\mathbb{R}^N$ structure of state space requires us to treat the spanning basis vectors $|\psi_k\rangle$ as static objects though they are the main dynamical objects in QM. The vectors that span a Hilbert space are static because there is a unique Hilbert space associated with each time t. However, the $\mathbb{R}^N$ picture of a static basis is useful for envisioning the time evolution of quantum states. Given

$$\hat{A}\big|a_k\big\rangle = a_k\big|a_k\big\rangle \quad , \quad \text{and} \qquad \big|\psi,t\big\rangle = \sum_{k=1}^{N} c_k(t)\big|a_k\big\rangle \quad , \tag{1.3.1}$$

one understands that $|\psi,t\rangle$ is a vector sweeping through the $\mathbb{R}^N$ spanned by $\{|a_k\rangle\}$. The $|a_k\rangle$ eigenbasis is the geometric spanning basis of the space of states written in that basis. Although a Hilbert space is technically the space of states at some constant time t, time evolution may be understood as a continuous evolution in state space. Time evolution described by a sweeping vector $|\psi,t\rangle$ is simplified by the unitarity constraint: the tip of $|\psi,t\rangle$ always lies on the N-dimensional unit sphere such that

$$\big\langle\psi,t\big|\psi,t\big\rangle = \sum_k c_k^\dagger(t)c_k(t) = 1 \quad , \tag{1.3.2}$$

where $c_k(t)$ is as in (1.3.1). The components of a vector in the $\{|a_k\rangle\}$ basis are written as $(c_1,c_2,...,c_N)$ so (1.3.2) defines a point on the unit sphere whose equation is $\sum x_k^2=1$. The *coordinates of state space* are such that the $x_k(t)$ Cartesian coordinates are replaced with the $c_k(t)$ coefficients in the expansion of $|\psi,t\rangle$. State space has this structure for geometric interpretation but quantum theory is not such that one refers to such things in practice. We will use it here to demonstrate compliance with vector and tensor transformation laws in a mathematically rigorous way. This will exceed the compliance usually demonstrated through the above bulleted axioms of a vector space.

1.3.2 The Vector Transformation Law

Vector notation is such that

$$\mathbf{x} = \sum_k a^k \hat{e}_k \qquad \Longrightarrow \qquad x^\mu = a^\mu \quad . \tag{1.3.3}$$

At first glance, we can tell that ordinary states and MCM states are vectors and tensors respectively from

$$|\psi\rangle = \sum_k a^k |a_k\rangle \qquad \Longrightarrow \qquad \psi^\mu = a^\mu \quad , \tag{1.3.4}$$

and its generalization as an MCM state

$$\big|\psi; \hat{e}_\mu\big\rangle = \sum_k a_k \big|a_k\big\rangle \hat{e}_\mu \qquad \Longrightarrow \qquad \psi_{\mu\nu} = a_\mu \hat{e}_\nu \quad . \tag{1.3.5}$$

A one index tensor is a vector and a vector with an extra index is a tensor. However, this is not a formal demonstration of the transformation law. A more formal statement of the law would be the following.

> For a unit vector $\hat{n}$ and an angle ϕ, let $\hat{R}(\hat{n}, \phi)$ be a rotation operator. Suppose $|\psi\rangle = |a\rangle + |b\rangle$. If $\hat{R}$ preserves the "angle" between $|a\rangle$ and $|b\rangle$, meaning that $\hat{R}|\psi\rangle = \hat{R}|a\rangle + \hat{R}|b\rangle$ is such that $\hat{R}|a\rangle$ and $\hat{R}|b\rangle$ are still orthogonal if $|a\rangle$ and $|b\rangle$ were orthogonal, then $|\psi\rangle$ transforms as a vector.

If two orthogonal objects belong to a vector space, then they will remain orthogonal under coordinate transformations. If ψ is not written in the position space representation, then the details become modestly more complicated because the rotation operator, which is only one example of a coordinate transformation, must pertain to the coordinates of state space. As it is usually understood, $\hat{n}$ indicates some spatial rotation axis in an $\mathbb{R}^3$ lab frame. It does not make sense to rotate a state around such an axis when the state is not written in the position basis. Instead, we must generalize to the case where $\hat{n}$ points in an abstract direction defined according to the $\{|a_k\rangle\}$ spanning states instead of the $\{\hat{x}, \hat{y}, \hat{z}\}$ basis that spans $\mathbb{R}^3$. Indeed, we must generalize to the case of arbitrary coordinate changes in state space. Consideration of rotations alone will not suffice for a rigorous demonstration.

As a thinking device, one might consider the 2D space of electron spin states

$$\hat{x} \equiv \hat{e}_1 = \big| \uparrow \big\rangle \quad , \quad \text{and} \qquad \hat{y} \equiv \hat{e}_2 = \big| \downarrow \big\rangle \quad . \tag{1.3.6}$$

These states transform as spinors under rotations of the lab frame (physical space), but they transform as vectors under rotations of state space. The state space spanned by these eigenstates is $\mathbb{R}^2$. The well known time evolution of these states is visualized as the tip of a vector moving on the unit circle.

To formally demonstrate the vector transformation law for a vector in $\mathbb{R}^N$, let x be a coordinate system in $\mathbb{R}^N$ and let there be a coordinate transformation

$$x' = f(x) \quad , \quad \text{such that} \qquad x'^{\mu} = T^{\mu}_{\nu} x^{\nu} \quad . \tag{1.3.7}$$

Let $\mathbf{v} = v^{\mu}$ be a vector in $\mathbb{R}^N$ written in the x coordinates. It follows that

$$\mathbf{v} = \sum_k^n x^k \hat{e}_k \qquad \Longrightarrow \qquad \begin{cases} \mathbf{v} = x^{\mu} \hat{e}_{\mu} \\ v^{\mu} = x^{\mu} \end{cases} \qquad . \tag{1.3.8}$$

$\mathbf{v}$ is anchored at the origin of the x coordinate system and its tip is at the point x. In conventional notation, the most general statement of the vector transformation law for vectors in $\mathbb{R}^N$ is

$$v'^{\mu} = v^{\nu} \frac{\partial x'^{\mu}}{\partial x^{\nu}} \quad , \tag{1.3.9}$$

where $\mathbf{v}' = v'^{\mu}$ is $\mathbf{v}$ written in the transformed x' coordinates. Following the form of (1.3.8), we may write

$$\mathbf{v}' = \sum_k^n x'^k \hat{e}'_k \qquad \Longrightarrow \qquad \begin{cases} \mathbf{v}' = x'^{\mu} \hat{e}'_{\mu} \\ v'^{\mu} = x'^{\mu} \end{cases} \qquad . \tag{1.3.10}$$

Taking the derivative of (1.3.7) with respect to x^{ν} gives

$$\frac{\partial x'^{\mu}}{\partial x^{\nu}} = T^{\mu}_{\nu} \quad . \tag{1.3.11}$$

Substitution into (1.3.9) gives

$$v'^{\mu} = v^{\nu} T^{\mu}_{\nu} \quad . \tag{1.3.12}$$

Substituting $v^{\nu} = x^{\nu}$, we obtain

$$v'^{\mu} = x^{\nu} T^{\mu}_{\nu} = x'^{\mu} \quad , \tag{1.3.13}$$

where the second equality follows from (1.3.7). The result agrees with (1.3.10). Therefore, the vector transformation law is satisfied by vectors in $\mathbb{R}^n$ under arbitrary coordinate transformations, as is obvious since our objects were taken as vectors to begin with.

1.3.3 Vector Transformations for Ordinary States

To demonstrate the transformation above with vector states in Hilbert space, we write the vector transformation law as

$$\psi'^{\mu} = \psi^{\nu}\frac{\partial x'^{\mu}}{\partial x^{\nu}} \quad . \tag{1.3.14}$$

In this case, it may not be obvious what are x and x', or what is meant by ψ and ψ'. Noting that a general state vector is written as

$$\big|\psi\big\rangle = \sum_k^N \alpha_k \big|a_k\big\rangle \qquad \Longrightarrow \qquad \psi^{\mu} = \alpha_{\mu} \quad ,^{1} \tag{1.3.15}$$

we see that $\mathbf{v}=\sum_k v_k\hat{e}_k$ implies $v_k\rightarrow\alpha_k$ and $\hat{e}_k\rightarrow|a_k\rangle$. What we have demonstrated as a coordinate transformation in the previous section will now be phrased as the familiar change of basis operation. The coordinate systems x and x' will be two different operator eigenbases. In (1.3.15), we have implicitly used $\hat{\mathsf{A}}|a_k\rangle = a_k|a_k\rangle$ to expand ψ in the eigenbasis of $\hat{\mathsf{A}}$. ψ' will be the expansion in another eigenbasis. To rewrite $|\psi\rangle$ in terms of the eigenstates of some other operator $\hat{\mathsf{B}}$ such that $\hat{\mathsf{B}}|b_k\rangle = b_k|b_k\rangle$, we insert the completeness relation

$$\mathbb{1} = \sum_j^N \big|b_j\big\rangle\big\langle b_j\big| \quad , \tag{1.3.16}$$

into (1.3.15). This yields

$$\big|\psi\big\rangle = \sum_k^N \alpha_k \mathbb{1}\big|a_k\big\rangle = \sum_k^N\sum_j^N \alpha_k\big|b_j\big\rangle\big\langle b_j\big|a_k\big\rangle \quad . \tag{1.3.17}$$

Now we obtain the coordinate transformation analogue

$$\beta_j = \sum_k^N \alpha_k\big\langle b_j\big|a_k\big\rangle \quad , \tag{1.3.18}$$

[1] Here we have intermingled tensor and matrix index notation, as is usual in physics. If desired, one might write α^{μ} so that the indices balance as $\psi^{\mu}=\alpha^{\mu}$. (1.3.15) follows a standard physical convention in which expansion coefficients are labeled with lower indices.

with which to write

$$\big|\psi\big\rangle=\sum_j^N\beta_j\big|b_j\big\rangle\quad .^1 \tag{1.3.19}$$

This is the expression for what we have called $\mathbf{v}'$ in the previous section. It is the same state vector written in another eigenbasis which is like another coordinate system in the geometric picture of state space. This is the ψ' appearing in (1.3.14). Switching from summation notation to matrix multiplication notation, (1.3.18) becomes

$$\beta_j=\alpha_kT_{jk}\quad , \tag{1.3.20}$$

for the transformation matrix whose elements are $T_{jk}=\langle b_j|a_k\rangle$. Notice that (1.3.20) is in the form of (1.3.7) with $x'^\mu\rightarrow\beta_j$ and $x^\nu\rightarrow\alpha_k$. Now that we have very clearly spelled out all of the details, we may write the vector transformation law for quantum state vectors, (1.3.14), as

$$\psi'^j=\psi^k\frac{\partial\beta_j}{\partial\alpha_k}\quad . \tag{1.3.21}$$

The derivative follows from (1.3.20) as

$$\psi'^j=\psi^k\frac{\partial}{\partial\alpha_k}\big(\alpha_kT_{jk}\big)=\psi^kT_{jk}\quad . \tag{1.3.22}$$

Substituting the j^{th} coefficient from (1.3.19) on the left, and the k^{th} coefficient from (1.3.15), we obtain

$$\beta_j=\alpha_kT_{jk}\quad , \tag{1.3.23}$$

which is true by (1.3.20). Now we have proven that vectors in state space transform exactly like vectors in coordinate space. Such functioning is not highly useful for QM as practiced but the result is valid.

1.3.4 Tensor Transformation of MCM States

For a two-index tensor, the tensor transformation law is

$$\psi'^{\mu\nu}=\psi^{\kappa\lambda}\frac{\partial x'^\mu}{\partial x^\kappa}\frac{\partial x'^\nu}{\partial x^\lambda}\quad . \tag{1.3.24}$$

Using the definition $\psi_{\mu\nu}=|\psi;\hat{e}_\nu\rangle=\psi_\mu\hat{e}_\nu$, we have already shown the that the μ index transforms correctly. The coordinates relevant to transformations of the other index

[1] The sum over k and j both go to N because change of basis operations should preserve the dimensionality of the Hilbert space.

are those specified by

$$\begin{aligned}
\psi \in \mathcal{A} \qquad &\Longleftrightarrow \qquad \big|\psi\big\rangle = \big|\psi;\hat{2}\big\rangle = \psi(x^i_-) \\
\psi \in \mathcal{H} \qquad &\Longleftrightarrow \qquad \big|\psi\big\rangle = \big|\psi;\hat{\pi}\big\rangle = \psi(x^i) \\
\psi \in \Omega \qquad &\Longleftrightarrow \qquad \big|\psi\big\rangle = \big|\psi;\hat{2}\big\rangle = \psi(x^i_+) \\
\psi \in \varnothing \qquad &\Longleftrightarrow \qquad \big|\psi\big\rangle = \big|\psi;\hat{i}\big\rangle = \psi(x^i_\varnothing) \quad .
\end{aligned} \tag{1.3.25}$$

Rather than general coordinate transformations, we will demonstrate coordinate transformations among the physical coordinates of $\mathcal{A}$, $\mathcal{H}$, and Ω (and possibly $\varnothing$.) Since the scope of transformations is limited, it will suffice to demonstrate a single case. We will use the transformation from the $\hat{\pi}$ coordinates to the $\hat{\Phi}$ coordinates so that x is the unprimed coordinate and x_+ is the primed coordinate.

Using the transformation operator $\hat{\mathsf{O}}$ (Section 1.2.1) rather than the transformation matrix T, we have

$$\hat{\mathsf{O}}_{\mathcal{H}\to\Omega}\, x\hat{\pi} = x_+\hat{\Phi} \quad . \tag{1.3.26}$$

Following the example for $\hat{\mathsf{O}}$ given by (1.2.22), we obtain

$$\hat{\mathsf{O}}_{\mathcal{H}\to\Omega}\, x\hat{\pi} = \hat{\mathsf{O}}_{\mathcal{H}\to\Omega}\left(\frac{\Phi}{\Phi}\, x\hat{\pi}\right) = \frac{\pi}{\Phi}\, x\hat{\Phi} \qquad \Longrightarrow \qquad x_+ = \frac{\pi}{\Phi}\, x \quad . \tag{1.3.27}$$

It follows that

$$\frac{\partial x^\nu_+}{\partial x^\lambda} = \frac{\pi}{\Phi}\,\delta^\nu_\lambda \quad , \quad \text{and} \qquad \frac{\pi}{\Phi}\,\delta^\nu_\lambda = T^\nu_\lambda \quad , \tag{1.3.28}$$

where T^ν_λ is the transformation matrix between the physical coordinates in $\mathcal{H}$ and those in Ω. One might write $\hat{\mathsf{O}}_{\mathcal{H}\to\Omega}=T^\nu_\lambda$. To verify the tensor transformation law for MCM states, it only remains to obtain the $\psi'^{\mu\nu} = |\psi;\hat{\Phi}\rangle$ state for comparison with (1.3.24):

$$\hat{\mathsf{O}}_{\mathcal{H}\to\Omega}\big|\psi;\hat{\pi}\big\rangle = \hat{\mathsf{O}}_{\mathcal{H}\to\Omega}\left(\frac{\Phi}{\Phi}\,\big|\psi\big\rangle\hat{\pi}\right) = \frac{\hat{\Phi}}{\Phi}\,\big|\psi\big\rangle\pi = \frac{\pi}{\Phi}\big|\psi;\hat{\Phi}\big\rangle \quad . \tag{1.3.29}$$

This demonstration for the ν index suffices to verify the tensor transformation law for MCM states.

1.4 MCM Spin Spaces

The proposed structure for MCM spin state space configurations [6] is such that states in $\mathcal{H}_0$ reference local elements of the unit cell or those in higher and lower levels of aleph. It was suggested in Section 1.2.4 that $\chi^4_\pm$ might be made complex in

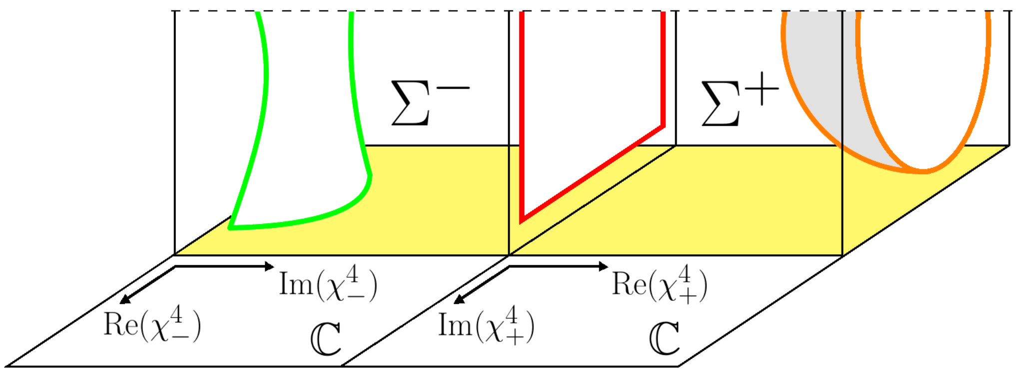

Figure 5: This figure demonstrates that we may take $\chi^4_\pm$ as complex variables whose real and imaginary parts span $\Sigma^\pm$ respectively.

the direction out of the page but mutually orthogonal and still orthogonal to x^i, as in Figure 5. In this section, we will suggest the same for x^0 and $x^0_\pm$: the chronological times in $\mathcal{H}$, $\mathcal{A}$, and Ω.

Spin-1/2 state space is canonically constructed as $L^2(\mathbb{R}^3)\otimes\mathbb{C}^2$ where $L^2(\mathbb{R}^3)$ is the spinless state space and $\mathbb{C}^2$ is a 2D complex vector space. In the MCM protocol, the spin-1/2 state space is constructed as

$$L^2(\mathbb{R}^3)\otimes\chi^4_{+\{0\}}\otimes\chi^4_{-\{0\}} \quad . \tag{1.4.1}$$

$\chi^4_{\pm\{0\}}$ are a pair of complex numbers whose respective real and imaginary parts are the $\chi^4_\pm\in\Sigma^\pm$ on the zeroth level of aleph. Across the unit cell, χ^4_+ and χ^4_- are uniquely real and imaginary but they are both complex when we take their mutually orthogonal transverse continuations onto $\mathbb{C}$. The spin-1 state space is canonically constructed as $L^2(\mathbb{R}^3)\otimes\mathbb{C}^3$ where $\mathbb{C}^3$ is a 3D complex vector space. In the MCM, we use

$$L^2(\mathbb{R}^3)\otimes x^0_{+\{0\}}\otimes x^0_{\{0\}}\otimes x^0_{-\{0\}} \quad , \tag{1.4.2}$$

where $\text{Im}(x^0_{\{k\}})$ is the x^0 coordinate in $\mathcal{H}_k$ generating the minus sign in the $\{-+++\}$ signature of Minkowski space. $x^0_{\pm\{k\}}$ are also understood to be complex.

For fermionic spin-$\frac{2N-1}{2}$ with $N>1$, usually $L^2(\mathbb{R}^3)\otimes\mathbb{C}^{2N}$, we take the tensor product of (1.4.1) with $\chi^4_{\pm\{k\}}$ on other levels of aleph, however many are needed to assemble the requisite spin degrees of freedom:

$$L^2(\mathbb{R}^3)\bigotimes_{k=0}^{N-1}\chi^4_{\pm\{k\}} \quad . \tag{1.4.3}$$

If allowing $\chi^4_{\pm\{k\}}$ to become complex is found to be too complicated or needlessly complicated, we might construct the spin-$\frac{2N-1}{2}$ state space as

$$L^2(\mathbb{R}^3)\bigotimes_{k=0}^{\frac{N-1}{2}}\left(\chi^4_{-\{k\}}\oplus\chi^4_{+\{k-1\}}\right)\otimes\left(\chi^4_{+\{k\}}\oplus\chi^4_{-\{k+1\}}\right)\quad , \tag{1.4.4}$$

where the χ^4 variants are strictly real or imaginary and $\mathbb{C}^N$ is formed from N such pairs. For $N=1$, this expression gives

$$L^2(\mathbb{R}^3)\otimes\left(\chi^4_{-\{0\}}\oplus\chi^4_{+\{-1\}}\right)\otimes\left(\chi^4_{+\{0\}}\oplus\chi^4_{-\{1\}}\right)\quad . \tag{1.4.5}$$

Each parenthetical pair of strictly real or imaginary $\chi^4_\pm$ constitutes one instance of $\mathbb{C}$. (1.4.5) generates the correct $L^2(\mathbb{R}^3)\otimes\mathbb{C}^2$ spin-1/2 state space without requiring $\chi^4_\pm$ to have simultaneous real and imaginary parts.

For bosonic spin-N, increasing N requires that we alternately add instances of $x^0_{\{k\}}$ and $x^0_{\pm\{k\}}$ for odd or even N but a regular recursion formula is not simply obtained. For spin-2, we have

$$L^2(\mathbb{R}^3)\otimes x^0_{-\{1\}}\otimes x^0_{+\{0\}}\otimes x^0_{\{0\}}\otimes x^0_{-\{0\}}\otimes x^0_{+\{-1\}}\quad . \tag{1.4.6}$$

For spin-3, we have

$$L^2(\mathbb{R}^3)\otimes x^0_{\{1\}}\otimes x^0_{-\{1\}}\otimes x^0_{+\{0\}}\otimes x^0_{\{0\}}\otimes x^0_{-\{0\}}\otimes x^0_{+\{-1\}}\otimes x^0_{\{-1\}}\quad , \tag{1.4.7}$$

and so forth. It is not immediately obvious what construction might avoid allowing x^0 to become complex in the manner of (1.4.4).

In Section 13, we will show a nice application of this spin space construction toward supersymmetry between bosons and fermions.

1.5 Maximum Action

Quantum and classical probabilities differ in that unmeasured, intermediate steps of quantum motion between two measurements cannot be inferred from those measurements.[1] If a twice-measured classical ball rolls down a ramp, it has a definite position at each instant during the motion. The motion can be inferred from either measurement, even if one looks away while the ball is rolling. The path is the one that minimized the action. The ball's wavefunction does not diffuse. It is always collapsed. For a quantum particle moving on some analogous energy landscape, the

[1] See Sections 2-4 in [67] or Section I.2 in [75] for a concise statement of classical and quantum probabilities.

position of the particle is not knowable while one is looking away. If a quantum ball is observed at a location with higher energy and then at one with lower energy, and in the absence of any intermediate measurements, it may not have followed the path which minimized the action. Indeed, the most common interpretation of QM is that a quantum particle does not follow *any* path between consecutive measurements. Between measurements, a position state is said to undergo *decoherence* [76] such that it evolves into an increasingly diffuse superposition of eigenstates conglomerated about the classical trajectory. Decoherence is the heatlike diffusion of probability amplitude given by the Schrödinger equation, a heat equation. When one looks, the wavefunction collapses. Contrary to the classical case, the wavefunction diffuses while one looks away. The longer one looks away from a quantum state, the more likely it is to be found away from the path of classical motion.

The main insight in Feynman's formulation of non-relativistic quantum mechanics [67] was to show that the probability amplitude for the particle having followed one path or another is a fuzzy distribution proportional to the action along each path. The classical trajectory minimizes the action so the probability amplitude is greatest along that path. The more a path fails to extremize the action, the less probable it is that the particle might be observed along that path.

The usual formulation of QM is such that nothing other than diffusion happens between two consecutive measurements A and B. The main purpose in writing $\hat{M}^3$ as three separate operations is to hard-code into the motion stops on Ω and $\mathcal{A}$ between successive $\mathcal{H}$ so as to increase the richness of possible dynamics. Though measurements can only be made in $\mathcal{H}$ (the universe), the MCM postulates by construction that there exists definite knowledge that the state was located on Ω and $\mathcal{A}$ between t_0 and t_1 corresponding to measurements A and B. Using intuitive notation such that $t_0 < t_\Omega < t_\mathcal{A} < t_1$, we know that MCM states "collapse" to $|\psi, t_\Omega; \hat{\Phi}\rangle$ and $|\psi, t_\mathcal{A}; \hat{2}\rangle$ between measurements A and B (corresponding to states $|\psi, t_0; \hat{\pi}^0\rangle$ and $|\psi, t_1; \hat{\pi}^1\rangle$.) Additional knowledge of the state at the intermediate times t_Ω and $t_\mathcal{A}$ is part of what is meant when it is said that $\hat{M}^3$ is purposed to make things more complicated than what is understood for ordinary operations in QM. At minimum, additional complexity is manifested by three separate time evolutions $\mathcal{H} \to \mathcal{A} \to \Omega \to \mathcal{H}$ where the sign convention for the arrow of time differs between Σ^+ and Σ^-. In Section 1.8.5, we will discuss an application in which Schrödinger evolution by negative time might implement a phase of wavefunction collapse following a phase of wavefunction diffusion in positive time.

It is a conjecture of the MCM that quantum and classical motions differ in the way that they satisfy the action principle. Classical motion minimizes action and quantum motion maximizes it. It is taken for granted that motion along any path totally within $\mathcal{H}$ must be associated with some finite action. Therefore, the path which leaves the universe ($\mathcal{H}$) to cross the unit cell is associated with infinite action. For the purposes of physics, what is usually called finite action may be defined as action less than some natural number of finite action increments: $n\hbar$ with $n\in\mathbb{N}$, for example. In the language of fractional distance analysis (Section 1.6) [2], a natural number of units of action is called an action in the neighborhood of the origin. By default, action in **the neighborhood of infinity** remains to characterize motion across the unit cell. The neighborhood of infinity may be characterized as the set of numbers in the form $\widehat{\infty}\pm b$ with $0<b<n$ for some $n\in\mathbb{N}$. Numbers in the form $\widehat{\infty}-b$ are finite numbers because they are less than infinity. (Notations for $\widehat{\infty}$ are developed in Section 1.6 and [2].) Thus, one is able to use such numbers to characterize motion across the unit cell without violating a physical convention prohibiting infinite or transfinite quantities of action. Action greater than infinity would allow superluminal motions, etc.

Any action in the neighborhood of infinity will be one which takes the state out of $\mathcal{H}$. Such an action makes an immediate appeal to the correspondence principle: when action is large compared to $\hbar$, motion should approach the classical motion. In other words, large action impedes the diffusion of the wavefunction. Thus, knowledge regarding states' definite location on the Ω- and $\mathcal{A}$-branes between sequential $\mathcal{H}$-branes is supported by the correspondence principle. Classical motion is characterized by definite knowledge of the path between A and B. In the example of the ball on a ramp, the action of the classical ball's motion is always large relative to $\hbar$ due to the ball's macro-scale mass. In QM, one often considers *large action* as the limit in which $\hbar\to 0$ but here we will consider $\mathcal{S}\to\widehat{\infty}$. The arithmetic of numbers in the neighborhood of infinity [2] is well-suited to the calculus of variations with variations in the form $\mathcal{S}=\widehat{\infty}\pm\delta\mathcal{S}$. To the contrary, $\mathcal{S}\to\infty$ is a prime example of the "infinities that blocked earlier theories" [69]. $\mathcal{S}=\infty$ is a mathematical non-starter for calculus. The study of maximum action has been historically impossible for this reason. For instance, Hamilton's stationary action principle requires any action extremum, big or small, but Feynman's thesis was titled "The Principle of Least Action in Quantum Mechanics" because greatest action was a non-starter at that time. A principle of greatest action in QM is presented here as a thesis awaiting completion, i.e.: the equations of motion given by $\hat{M}^3$ should satisfy the greatest action principle.

In the $\hbar\to 0$ limit, or in the $\mathcal{S}\to\widehat{\infty}$ limit, one obtains a classical motion identically. Identical classical motion during transit of the unit cell is not consistent with the structure of the MCM because KKT requires that the 5D Ricci tensor R_{AB} must vanish in the bulk of $\Sigma^{\pm}$. If a particle with mass and energy follows a classical path across $\Sigma^{\pm}$, then $R_{AB}^{\pm} \neq 0$ and the structure of the MCM will seem to collapse in self-contradiction.[1] It is required and that a quantum of matter-energy should not be found with a definite position inside the bulk. Therefore, an appeal is made to finite action in the neighborhood of infinity. Unlike $\mathcal{S}=\widehat{\infty}$, finite action in the form $\widehat{\infty}-\delta\mathcal{S}$ with $\delta\mathcal{S}>0$ may not require total wavefunction collapse within the bulk so the utility of such an action toward preserving the structure of KKT must be examined.

A pseudo-classical path of totally classical motion would be the one along which $\mathcal{S}=\widehat{\infty}$. This is an extremum of the action and motion along this singular path cannot be quantum. However, since measurements are not made in the bulk, we may send a particle across the unit cell by *all paths* whose actions are $\mathcal{S}=\widehat{\infty}-\delta\mathcal{S}$. Quantum states may transit the unit cell without taking any one explicit path to force a non-vanishing Ricci tensor. In that case, it will remain to demonstrate that a non-zero probability amplitude in the bulk of $\Sigma^{\pm}$ is still consistent with an $R_{AB}=0$ solution. The existence of non-trivial $R_{AB}=0$ solutions such as gravitational radiation support the idea that a probability density for motion near the infinite action path can be consistent with $R_{AB}=0$. One might even connect the principle of maximum action to states passing from one $\mathcal{H}$-brane to another as gravitational waves written as perturbations in the 5D KKT metric. Furthermore, definite location on the Ω- and $\mathcal{A}$-branes must also be reconciled with the vanishing Ricci tensor. If $|\psi,t_\Omega;\hat{\Phi}\rangle$ and $|\psi,t_\mathcal{A};\hat{2}\rangle$ are not position eigenstates, then we might appeal to the same indeterminacy of the path in the bulk to avoid a Ricci tensor violation. If they are position eigenstates, or if any other issue arises, one might preserve KKT in the bulk of $\Sigma^{\pm}$ by separating Ω and $\mathcal{A}$ as unincluded boundaries, as $\mathcal{H}$ is an unincluded boundary. We will say more about that possibility in Section 4. Specifically, we will discuss the case for colocating Ω and $\mathcal{A}$ at $\varnothing$.

1.6 Fractional Distance and Levels of Aleph

The labeled branes of the unit cell are separated by finite distance in the abstract coordinates. To avoid mutual interactions, and specifically to avoid gravitation between branes, early work in the MCM sought to place $\mathcal{A}$, $\mathcal{H}$, and Ω at infinite distances with respect to one another. Due to the infinite range of the gravitational force, finite physical distance between branes would suggest gravitational collapse of the overall lattice of all unit cells. On the other hand, infinite distance is said to be unphysical.

[1] The full restrictions of KKT require in-depth analysis, as in Section 17. It is the preliminary understanding that position eigenstates for massive particles in the bulk of $\Sigma^{\pm}$ are not allowed.

One exciting utility for fractional distance analysis [2] is that the gravitational interaction goes to zero across any *finite* distance in the neighborhood of infinity. Indeed, the MCM requirement for branes separated by analytically tractable distances across which gravitation goes to zero was the progenitor of the ideation which led to the discovery of fractional distance and an interesting corollary regarding the Riemann hypothesis [2, 46–48].

1.6.1 Infinity Hat

The main output of the inquiry into fractional distance was a new algebraic object $\widehat{\infty}$. It is called **algebraic infinity** to distinguish it from ∞, called **geometric infinity**. Informally, $\widehat{\infty}$ was already in wide use in physics before it was formalized in [2]. In QFT for example, one often writes the integral over all of spacetime as

$$\int d^4x = \int d^3x \int dx^0 = VT \quad , \tag{1.6.1}$$

where V is the volume of space and T is an infinite amount of time which can cancel with another T somewhere else via $T/T = 1$. This common physical method for dealing with infinity is replicated with $T = \widehat{\infty}$ and the arithmetic axioms for numbers in the neighborhood of infinity [2]. The main difference between $\widehat{\infty}$ and ∞ is that the latter has properties of additive and multiplicative absorption

$$x \in \mathbb{R} \quad \implies \quad \begin{cases} x + \infty = \infty \\ x \times \infty = \infty \end{cases} \quad , \tag{1.6.2}$$

but $\widehat{\infty}$ does not have those properties. Its main algebraic properties are

$$\begin{aligned} \widehat{\infty} - \widehat{\infty} &= 0 \\ \frac{\widehat{\infty}}{\widehat{\infty}} = \frac{\widehat{\infty}}{\infty} &= 1 \\ 0 \times \widehat{\infty} &= 0 \\ |\widehat{\infty}| &= \infty \quad . \end{aligned} \tag{1.6.3}$$

See [2] for more details regarding the arithmetic of $\widehat{\infty}$.

There is a theorem in [2] (Main Theorem 3.2.6) proving that some $x \in \mathbb{R}$ are greater than any $n \in \mathbb{N}$. Consequently, there exist some $x \in \mathbb{R}$ having greater than zero **fractional distance** with respect to infinity. The number $\aleph_{\mathcal{X}}$ defined by

$$\forall \mathcal{X} \in (0,1) \quad \exists \aleph_{\mathcal{X}} \in \mathbb{R} \quad , \quad \text{such that} \qquad \frac{\aleph_{\mathcal{X}}}{\infty} = \mathcal{X} \quad , \tag{1.6.4}$$

is said to have fractional distance $\mathcal{X}$ (with respect to infinity) because $\aleph_{\mathcal{X}}/\infty = \mathcal{X}$. The subset of $\mathbb{R}$ containing numbers having fractional distance $\mathcal{X}$ is labeled $\mathbb{R}_{\mathcal{X}}$, i.e.:

$$x \in \mathbb{R}_{\mathcal{X}} \qquad \Longrightarrow \qquad \frac{x}{\infty} = \mathcal{X} \quad . \tag{1.6.5}$$

Building on these definitions, we may write

$$\begin{aligned} \mathbb{R}_0 &= \{\, x \;\big|\; -n < x < n \quad \text{for some} \quad n \in \mathbb{N} \,\} \\ \mathbb{R}_{\mathcal{X}} &= \big\{\, \aleph_{\mathcal{X}} + b \;\big|\; b \in \mathbb{R}_0 \,\big\} \\ \mathbb{R}_1 &= \big\{\, \widehat{\infty} - b \;\big|\; b \in \mathbb{R}_0^+ \,\big\} \quad . \end{aligned} \tag{1.6.6}$$

$\mathbb{R}_0$ is called the neighborhood of the origin. As the set of all real numbers less than some natural number (and greater than some negative natural number), every $x \in \mathbb{R}_0$ has zero fractional distance. When $\mathcal{X} \in (0,1)$, $\mathbb{R}_{\mathcal{X}}$ is called an intermediate neighborhood of infinity. $\mathbb{R}_1$ is called the maximal neighborhood of infinity. There is more than one real number in each neighborhood because

$$\left.\begin{aligned} \frac{\aleph_{\mathcal{X}}}{\infty} &= \mathcal{X} \\ \frac{b}{\infty} &= 0 \end{aligned}\right\} \qquad \Longrightarrow \qquad \frac{\aleph_{\mathcal{X}} + b}{\infty} = \frac{\aleph_{\mathcal{X}}}{\infty} + \frac{b}{\infty} = \mathcal{X} + 0 = \mathcal{X} \quad . \tag{1.6.7}$$

The positive-definite, *arithmatic* neighborhood of infinity is

$$\widehat{\mathbb{R}} = \mathbb{R}_1 \bigcup_{\mathcal{X} \in (0,1)} \mathbb{R}_{\mathcal{X}}. \tag{1.6.8}$$

We will discuss additional numbers in the neighborhood of infinity called *non-arithmatic* numbers in Section 1.6.6. The big and little parts of a real number are

$$\text{Big}(\aleph_{\mathcal{X}} + b) = \aleph_{\mathcal{X}} \quad , \quad \text{and} \qquad \text{Lit}(\aleph_{\mathcal{X}} + b) = b \quad . \tag{1.6.9}$$

1.6.2 Levels of Aleph

Prior to the invention of $\widehat{\infty}$, levels of aleph were introduced in [70]. The theoretical framework for levels of aleph is the area of the MCM in which the most technical progress has been made. Levels of aleph are now associated with successive neigh-

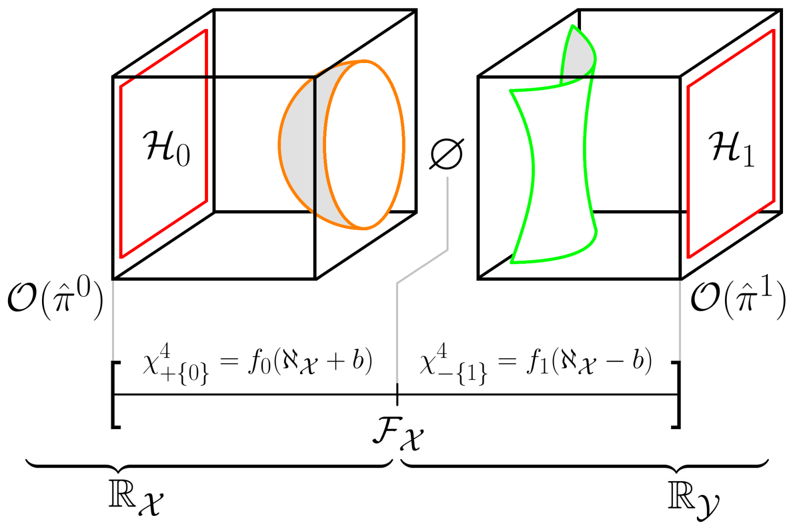

Figure 6: Relative to some absolute origin of $\mathbb{R}$ (not pictured), the origin of coordinates $\mathcal{O}(\hat{\pi}^n)$ on the n^{th} level of aleph is placed at $\aleph_\mathcal{X}$. $\aleph_\mathcal{X}$ is the midpoint of the interval representation of $\mathbb{R}_\mathcal{X}$. After operation with $\hat{M}^3$, the observer resides on a higher level of aleph whose origin of coordinates is placed at $\aleph_\mathcal{Y}$. $\mathcal{F}_\mathcal{X}$ is an *immeasurable number* described in Section 1.6.6.

borhoods of fractional distance.

Each unit cell is said to be on its own level of aleph. Recalling that we have placed $\mathcal{A}$ at $\chi^4_- = -\varphi$, $\mathcal{H}$ at $\lim \chi^4_\pm \to 0$, and Ω at $\chi^4_+ = \Phi$, a first approximation to a formal definition for each unit cell being on its own level of aleph is that there exists a bijection between some $\mathbb{R}_\mathcal{X} \backslash \aleph_\mathcal{X}$ (physical coordinates) and the chirological interval $(-\varphi, 0) \cup (0, \Phi)$ (abstract coordinates) around a corresponding instance of $\mathcal{H}$. Since $\chi^4_\pm = 0$ is not defined, bijection requires that we remove one number from the $\mathbb{R}_\mathcal{X}$ codomain. By removing $\aleph_\mathcal{X}$ and choosing $b > 0$, one obtains two separate bijections between $\aleph_\mathcal{X} - b$ and χ^4_-, and between $\aleph_\mathcal{X} + b$ and χ^4_+, as in Figure 6. When successive $\mathcal{H}$-branes are on successive levels of aleph, any two instances of $\mathcal{H}$ are automatically separated by a physical distance greater than any natural number of meters.[1] This follows because any $n \in \mathbb{N}$ and $(\aleph_\mathcal{X} + b) \in \mathbb{R}_\mathcal{X}$ are such that $(\aleph_\mathcal{X} + b + n)$ is still in $\mathbb{R}_\mathcal{X}$. Therefore, the forward $\mathcal{H}$-brane must be advanced in the χ^4 direction by greater than a natural number of physical distance units.

In the pure mathematical analysis of fractional distance appearing in [2], the metric along $\mathbb{R}$ was taken as the Euclidean metric. However, the application in the MCM for successive levels of aleph to exist on different scales requires a metric such that

[1] There does not exist a clear requirement for an $x^4_\pm$ physical coordinate but we presently discuss the case.

$\text{len}(\mathbb{R}_\mathcal{X}) \neq \text{len}(\mathbb{R}_\mathcal{Y})$ when $\mathcal{X} \neq \mathcal{Y}$. Aside from the irrational, non-unit magnitude scale factor $2\pi\Phi$ inherent to $\hat{M}^3|\psi;\hat{\pi}^0\rangle = 2\pi\Phi|\psi;\hat{\pi}^1\rangle$, we might use the $\widehat{\infty}$ notation to implement a change of scale such that the length of one neighborhood is infinitely great or small with respect to another. This additional scale would be implicit in the exponent on $\hat{\pi}^k$ that enumerates levels of aleph.

1.6.3 Gravitational Potential Energy

One way to avoid gravitation between branes is to suppose that there does not exist any physical counterpart to the abstract χ^4 coordinate on the fifth dimension. If the gravitational potential energy $U \neq U(\chi^4)$, then there is no Newtonian gravitation across $\Sigma^\pm$. However, it may be desirable to define a physical distance between branes in addition to the abstract distance. In that case, consider branes $\mathcal{H}_1$ and $\mathcal{H}_2$ as masses m_1 and m_2 separated by a real-valued distance $\mathbf{r}$ such that $|\mathbf{r}| \notin \mathbb{R}_0$. Let $\mathbf{r} = \aleph_\mathcal{X}\hat{r}$ with $\mathcal{X} > 0$ so the gravitational potential energy is

$$U(r) = -\frac{Gm_1m_2}{\aleph_\mathcal{X}} = -\frac{Gm_1m_2}{\mathcal{X}}\frac{1}{\widehat{\infty}} = \frac{0}{\mathcal{X}} = 0 \quad . \tag{1.6.10}$$

It follows that $\mathcal{H}$-branes will not mutually gravitate if G, m_1, and m_2 remain in the neighborhood of the origin.

We have not yet considered that change of scale might refer to quantities other than distances in the metric. If the scale of the level of aleph associated with $\mathcal{H}_2$ is such that $m_2 \notin \mathbb{R}_0$, then a non-zero gravitational energy will result, even across separations in the neighborhood of infinity. Given $m_2 = \aleph_\mathcal{Y} \in \mathbb{R}_\mathcal{Y}$, we have

$$U(r) = -\frac{Gm_1\aleph_\mathcal{Y}}{\aleph_\mathcal{X}} = -\frac{Gm_1\mathcal{Y}}{\mathcal{X}} \quad . \tag{1.6.11}$$

Mass in the neighborhood of infinity must be associated with curvature of spacetime in the neighborhood of infinity, indicating a likely singularity. If levels of aleph change the mass scale, one might conceive of an adjacent higher level of aleph as existing *within*, rather than beyond, the $\varnothing$ singularity that separates unit cells.

If non-zero gravitational energy is present between branes, we might consider the spin-1/2 matter particle interpretation of MCM universes to make an appeal to Pauli exclusion degeneracy pressure. This pressure will offset gravitational collapse and it may be important in the lattice whose branes are the standard model fermions. A simpler explanation for avoiding gravitational collapse may be that the Newtonian force still vanishes for m_2 in the neighborhood of infinity if one assumes an intuitive arithmetic:

$$\mathbf{F} = \frac{Gm_1m_2}{r^2}\hat{r} = \frac{Gm_1\aleph_{\mathcal{Y}}}{\aleph_{\mathcal{X}}^2}\hat{r} \quad \propto \quad \frac{1}{\widehat{\infty}} \quad . \tag{1.6.12}$$

To avoid a vanishing Newtonian force, one would have to scale Newton's constant G to the higher level of aleph as well. In that case, m_1 is the only remaining quantity (aside from $\hat{t}$) on the lower level of aleph and $\mathbf{F}$ mimics the force on an infinitesimal mass. In effect, we have rescaled the big and little parts of a real number as a little part and an infinitesimal part. While infinitesimal masses are not used in Newtonian gravitation, *only* infinitesimal test masses follow the geodesics usually derived in GR. All other masses will have backreaction that pushes them off of stationary geodesics.

1.6.4 Arithmetic in the Neighborhood of Infinity

The well known rules of arithmetic for numbers in the neighborhood of the origin are such that multiplication and division are mutually associative, e.g.:

$$x, y, z \in \mathbb{R}_0 \qquad \Longrightarrow \qquad x \times \left(\frac{y}{z}\right) = \left(\frac{x}{z}\right) \times y \quad . \tag{1.6.13}$$

Arithmetic for $\widehat{\infty}$ and other numbers with non-vanishing fractional distance requires that division and multiplication are not mutually associative in all cases [2], e.g.:

$$x, y, z \in \mathbb{R} \qquad \not\Longrightarrow \qquad x \times \left(\frac{y}{z}\right) = \left(\frac{x}{z}\right) \times y \quad . \tag{1.6.14}$$

Consequently, division is not identically multiplication by an inverse. Rather, division is a separate operation. Under the usual rules for associative arithmetic in the neighborhood of the origin, we might write for some $b \neq 0$

$$\aleph_{\mathcal{X}} + b = \mathcal{X}\,\widehat{\infty} + b\,\frac{\widehat{\infty}}{\widehat{\infty}} = \left(\mathcal{X} + \frac{b}{\widehat{\infty}}\right)\widehat{\infty} = (\mathcal{X} + 0)\,\widehat{\infty} = \aleph_{\mathcal{X}} \quad . \tag{1.6.15}$$

This implies $b = 0$, a contradiction. When associativity is not taken for granted, the manipulation in (1.6.15) stalls at the second step. It is not possible to pull out a factor of $\widehat{\infty}$ to form the parenthetical expression

$$\mathcal{X}\,\widehat{\infty} + b\,\frac{\widehat{\infty}}{\widehat{\infty}} \quad \longrightarrow \quad \left(\mathcal{X} + \frac{b}{\widehat{\infty}}\right)\widehat{\infty} \quad , \tag{1.6.16}$$

because that makes an appeal to associativity, i.e.:

$$\left(\frac{\widehat{\infty}}{\widehat{\infty}}\right) \times 1 = \left(\frac{1}{\widehat{\infty}}\right) \times \widehat{\infty} \quad . \tag{1.6.17}$$

The contradiction in (1.6.15) is avoided because (1.6.14) says that (1.6.17) is not implied. If associativity were allowed, we might manipulate (1.6.17) as

$$1 = \frac{\widehat{\infty}}{\widehat{\infty}} = \widehat{\infty} \times \frac{1}{\widehat{\infty}} = \widehat{\infty} \times 0 = 0 \quad . \tag{1.6.18}$$

Assuming associativity has produced another contradiction.

In the neighborhood of the origin, arithmetic operations are as they are usually understood. In (1.6.10), we were able to pull $Gm_1m_2/\mathcal{X}$ out of the fraction with $\widehat{\infty}$ because $G, m_1, m_2, \mathcal{X} \in \mathbb{R}_0$. Since $\hat{M}^3$ moves states across levels of aleph, a formal equation for $\hat{M}^3$ may exceed that which can be stated using arithmetic only in the neighborhood of the origin. Acknowledging that Dirac kets are only a concise notation for states' more complicated analytical representations, the requirement that $\hat{M}^3$ changes the level of aleph may require that the analytical expression for $\hat{M}^3|\psi;\hat{\pi}^0\rangle = c|\psi;\hat{\pi}^1\rangle$ diverges in the neighborhood of the origin. This would follow from infinite relative scale between successive unit cells.

1.6.5 Reference Frames on Levels of Aleph

The $\hat{M}^3$ operator sends ψ to the next higher level of aleph. Although that operator is non-unitary, the probability interpretation is restored by a translation of the observer's frame of reference onto the corresponding level of aleph, or into the corresponding unit cell with a given scale. Having better defined what a level of aleph is, now we may better clarify the what is meant by translation onto a higher level of aleph.

The constants 2, π, and Φ that we have used in $\hat{M}^3|\psi;\hat{\pi}^0\rangle = 2\pi\Phi|\psi;\hat{\pi}^1\rangle$ all belong to $\mathbb{R}_0$. Assuming $|\psi;\hat{\pi}^0\rangle$ is valued in the neighborhood of the origin—this follows from $\langle\psi|\psi\rangle = 1$ when we make accommodations for $\mathbb{C}$—multiplication by another number in the neighborhood of the origin such as $2\pi\Phi$ cannot yield a number in the neighborhood of infinity. The product of any two natural numbers is still less than another natural number so the non-unit scalar constant $2\pi\Phi$ is not sufficient to alter the fractional distance of $|\psi;\hat{\pi}^0\rangle$. Instead, the exponent on $\hat{\pi}$ should denote the scale of a given level of aleph relative to that in $\mathcal{H}_0$ labeled with $\hat{\pi}^0$.

If the unit cell of measurement A is in the $\mathbb{R}_\mathcal{X}$ neighborhood, then the unit cell of measurement B belongs to the sequentially greater $\mathbb{R}_\mathcal{Y}$ neighborhood. Since $\mathcal{X}$ and $\mathcal{Y}$ belong to a continuum $(0,1) \subset \mathbb{R}_0$, there is some nuance which must be resolved before we may label sequential neighborhoods as integer-valued levels of aleph. The countable enumeration of an uncountable set is not possible, in general.[1] The resolution to

[1] Treatment of paradoxical issues pertaining to the countable enumeration of an uncountable set may be found in

this problem comes through a physical treatment of the observer's reference frame. In general, the observer only knows about $\mathcal{H}$ and has no way to measure χ^4 relative to some absolute origin not in $\mathcal{H}$. In the absence of any information that might be used to calculate an absolute distance fraction $\mathcal{X}$, we introduce a convention such that the observer's current level of aleph is always $\hat{\pi}^0$. The normalization of all other quantities against this convention is what is meant by translation of the observer's frame of reference onto a new level of aleph. When the observer is on a given level of aleph corresponding to some $\mathbb{R}_\mathcal{X}$ neighborhood, the observer's origin of coordinates is placed at $\aleph_\mathcal{X}$. The level of aleph corresponding to the local $\mathbb{R}_\mathcal{X}$ neighborhood is the neighborhood of the origin in the observer's coordinates (Figure 6). After operation with $\hat{M}^3$, the observer at measurement B must redefine his coordinate system so that measurement A was taken on the $\hat{\pi}^{-1}$ level of aleph in a lower neighborhood of fractional distance. When $\hat{M}^3$ sends a state to the next higher level, there is no requirement to determine a $\mathcal{Y}$ that is the least real number greater than $\mathcal{X}$.[1] Thus, we avoid any problem pertaining to the paradoxical enumeration of uncountable objects by countable integers because distance fractions such as $\mathcal{X}$ and $\mathcal{Y}$ are not observable.

While an observer has no way to calculate $\mathcal{X}$ or $\mathcal{Y}$ relative to an absolute origin (which physics suggests should not exist), he does have information about the number of measurements he has taken. Such measurements are easily and properly labeled with integers. When we introduce a convention such that the level of aleph is regularized by defining the observer's origin of coordinates at the $\aleph_\mathcal{X}$ specified by $\hat{\pi}^n$ (Figure 6), the scale of the coordinates must also be regularized so that the probability interpretation of the wavefunction is restored after non-unitary evolution. Redefinition of the observer's coordinate system on the higher level of aleph is not only a translation, it is also a change of scale. These mechanisms and their details require further clarifications.[2]

1.6.6 Immeasurable Numbers

Another discovery in fractional distance analysis was the set $\mathbb{F}$ containing all **immeasurable real numbers**, also called **non-arithmatic real numbers**. Given

$$\mathcal{X} \neq \mathcal{Y} \quad \Longrightarrow \quad \mathbb{R}_\mathcal{X} \cap \mathbb{R}_\mathcal{Y} = \emptyset \ , \tag{1.6.19}$$

Section 7 of [2].

[1]Although real analysis has suggested previously that there cannot exist a least real number greater than another real number (or a least positive real number), fractional distance analysis seems to suggest that such numbers should exist. These issues are treated in Section 7 of [2].

[2]Normalization of the observer's new frame in $\mathbb{R}_\mathcal{X}$ back to $\mathbb{R}_0$ is such that numbers are altered as $(\aleph_\mathcal{X} \pm b) \to \pm b$. The positive-definite property of $x \in \mathbb{R}_\mathcal{X}$ is lost. Therefore, we might associate a reversed time arrow along χ^4_- with the property of negative numbers to increase in magnitude in the opposite direction to the increase of positive numbers.

meaning different neighborhoods of fractional distance do not intersect, the interval $\mathbb{R}=(-\infty,\infty)$ can be simply connected only if there exist real numbers not in any neighborhood of fractional distance. These are the non-arithmatic numbers $\mathcal{F}_\mathcal{X}\in\mathbb{F}$ such that $\mathcal{F}_\mathcal{X}$ is the least upper bound of the open set $\mathbb{R}_\mathcal{X}$. Previously in the history of analysis, irrational numbers were introduced to *complete* intervals of rationals. The immeasurables are introduced for the same purpose. Immeasurables complete the disconnected neighborhoods of infinity in the way that irrationals complete the disconnected rationals [2].

If the various piecewise $\chi^4_\pm$ and $\chi^4_\varnothing$ are concatenated to make a smooth curve from $\mathcal{H}_k$ to $\mathcal{H}_{k+1}$ in one affine parameter, call it χ^4, then the location of $\varnothing$ along that curve is given by some $\chi^4\in\mathbb{F}$, as in Figure 6. In other words, if the neighborhood of χ^4 around $\mathcal{H}_k$ is parameterized as $\mathbb{R}_\mathcal{X}$, then the higher level of aleph on the far side of $\varnothing$ is a neighborhood of greater fractional distance $\mathbb{R}_\mathcal{Y}$ such that $\mathcal{Y}>\mathcal{X}$. $\mathbb{R}_\mathcal{X}$ spans the interval of χ^4 on one level of aleph and $\mathbb{R}_\mathcal{Y}$ spans it on the next level. Figure 6 shows the relative arrangements. In the χ^4 parameterization of the path between two $\mathcal{H}$-branes, $\varnothing$ becomes a topological obstruction because $\chi^4\in\mathbb{F}$ is a non-arithmatic number.[1] Arithmetic is not defined in the usual way for such numbers but the value $\chi^4=\mathcal{F}_\mathcal{X}$ is a hard-coded topological boundary condition. Waves which time evolve in χ^4 cannot be simply transmitted through $\chi^4=\mathcal{F}_\mathcal{X}$. In this way, $\varnothing$ is similar to the topological obstruction at $\mathcal{H}$.

$\mathcal{H}$ and $\varnothing$ must function as topological obstructions to separate the KK theories in $\Sigma^\pm$. Recall that the MCM introduces two disconnected 5D metrics, each containing an EM potential 4-vector and a dual 4-vector. The extra pair of potential vectors is meant to avoid a requirement of KKT that all solutions must be ones in which the EM field strength tensor vanishes. The MCM workaround requires the mutual topological isolation of $\Sigma^\pm$. This is achieved with $\mathcal{H}$ placed at undefined $\chi^4_\pm=0$ and $\varnothing$ placed at $\mathcal{F}_\mathcal{X}\in\mathbb{F}$ for which normal arithmetic is not defined.

It has been supposed that the topological discrepancy between the $\Sigma^\pm$ metric signatures might be assigned to a phase acquired in a process akin to specular optical reflection from a singularity at $\varnothing$ [71]. Phase shifted optical reflection would be associated with gravitational transmission through a black hole/white hole pair in the $\varnothing$-brane.[2] Overall, the manner of forward connection from Σ^+ to Σ^- is prominent among the unresolved issues in the MCM, and in fractional distance analysis. To wit, it was not uniquely determined in [2] whether $\mathbb{F}$ is a set of disconnected points or

[1] Non-arithmatic numbers are motivated, defined, and discussed in Section 7.5 of [2].

[2] In the physical metric, $\varnothing$ will be associated with the high curvature limits of de Sitter and anti-de Sitter space and must, therefore, be a topological singularity of infinite curvature, or curvature in the neighborhood of infinity, in the physical coordinates.

disconnected intervals. It was assumed for simplicity that the $\mathcal{F}_\mathcal{X}$ are single numbers but they may be intervals of numbers. In an exactly congruent problem, the MCM has not yet determined whether $\mathcal{A}$ and Ω are separated by an interval, a point, or if their union is the object that we have labeled $\varnothing$.[1] Referring again to Figure 6, the thinking that branes should not mutually gravitate suggests that $\mathcal{A}$ and Ω should lie at (or in) $\mathcal{F}_\mathcal{X}$ relative to $\mathcal{H}$ but it is not determined if an interval should separate them. Such open questions regarding Ω, $\varnothing$, and $\mathcal{A}$ are treated independently in Section 4.

Strong congruence between fractional distance analysis and the MCM is further evidence that the latter is physically robust. In pure mathematical analysis [2], a paradox was suggested such that there should exist a least positive real number, or a least real number greater than another real number [2]. In the arena of physics, there is no such paradox because the absence of an absolute origin makes it impossible for an observer to compute physically meaningful *absolute* distance fractions. The absence of any absolute reference frame has been known at least since the time of Galileo. Indeed, coordinate transformations between arbitrary coordinate origins are called *Galilean* transformations. The lack of any absolute reference frame is integral to Einstein's theory of relativistic Lorentz transformations as well. Even questions about Mach's principle that escape description in GR refer to the same lack of an absolute reference frame [77]. Furthermore, the mathematical analysis of fractional distance left an open question regarding whether successive neighborhoods of fractional distance are separated by single numbers or intervals of numbers. This question is exactly mirrored in the issue of the forward connection of Σ^+ into Σ^- on a higher level of aleph. The main qualitative issues raised in the physical analysis were the main quantitative issues discovered in the mathematical analysis. This identical overlap between physics and an only-tangentially related exercise in real analysis is good evidence that the MCM is a robust physical theory. The prior precedent of uncanny historical overlap between analysis and physics is good evidence that $\hat{M}^3$ can be formalized at the level suggested in this paper.

1.6.7 The Big Exponential Function

Quantum states are most often represented as sums of exponential functions. The following modification to the exponential function was posed as an analytical structure on which one might differentiate representations of $|\psi;\hat{\pi}^k\rangle$ and $|\psi;\hat{\pi}^j\rangle$ when $j \neq k$. There is no such ready structure in the usual expression for the exponential function.

[1] In the convention where the union of $\mathcal{A}$ and Ω is identified with $\varnothing$, these branes would become unincluded boundaries of $\Sigma^{\pm}$.

In [70] we posed

$$e^{ikx}=\sum_{n=0}^{\infty}\frac{\big(ikx\big)^n}{n!}=\sum_{0}^{\aleph_0}\frac{\big(ikx\big)^n}{n!}+\sum_{\aleph_0}^{\aleph_\infty}\frac{\big(ikx\big)^n}{n!}+\sum_{\aleph_\infty}^{\aleph_{\infty\infty}}\frac{\big(ikx\big)^n}{n!}+...\quad,\tag{1.6.20}$$

where each sum over aleph pertains to a level of aleph. This early modification to e^x has been formalized subsequently as **the big exponential function** E^x [2]. In the current notation, e^x retains it usual meaning as a sum over $n\in\mathbb{N}$. Fractional distance is such that every natural number belongs to the neighborhood of the origin so it was supposed that the infinite sum in the exponential function might be expanded to include more than a natural number of terms. Given $\mathbb{N}_0\equiv\mathbb{N}\subset\mathbb{R}_0$, we define a new set $\mathbb{N}_\infty$ consisting of the natural numbers and their analogues in every neighborhood of fractional distance. Using $\mathbb{N}_\infty$, the big exponential function is

$$E^{ikx}=\sum_{n\in\mathbb{N}_\infty}\frac{\big(ikx\big)^n}{n!}=\sum_{n\in\mathbb{N}_0}\frac{\big(ikx\big)^n}{n!}+\sum_{n\in\mathbb{N}_{\mathcal{X}_1}}\frac{\big(ikx\big)^n}{n!}+\sum_{n\in\mathbb{N}_{\mathcal{X}_2}}\frac{\big(ikx\big)^n}{n!}+...\quad.\tag{1.6.21}$$

By the property

$$\left.\begin{aligned}&x\in\mathbb{R}_0\\&y\in\mathbb{R}_{\mathcal{Y}}\\&\mathcal{Y}>0\end{aligned}\right\}\quad\Longrightarrow\quad\frac{x}{y}=0\quad,\tag{1.6.22}$$

given in [2], it follows that kx in the neighborhood of the origin implies that all but the first sum in (1.6.20) will vanish. The sums over $n\notin\mathbb{N}_0$ in (1.6.21) vanish for the same reason when $kx\in\mathbb{R}_0$. It is proven in [2] (Theorem 6.2.5) that $E^x=e^x$ when $x\in\mathbb{R}_0$ but the big exponential function is not identically equal to e^x when $kx\notin\mathbb{R}_0$. Along with the new rules for arithmetic in the neighborhood of infinity, this function provides a tool for new methods in physics.

Spacelike and timelike coordinate separations often appear in the argument of the exponential function. The expression $\exp\{i[\mathbf{k}\cdot(\mathbf{x}_2-\mathbf{x}_1)-\omega(t_2-t_1)]\}$ is common enough. Therefore, one utility for E^x should be for the specification of wavefunctions on different levels of aleph such that $\Delta\mathbf{x}$ and Δt or their chirological analogues should be quantities with non-vanishing fractional distance. Given a wavefunction $|\psi,\hat{\pi}^n\rangle$, the $\hat{\pi}^n$ object might act as a window function—a Kronecker δ analogue—selecting only the sum over the $\mathbb{N}_{\mathcal{X}}$ corresponding to the k^{th} level of aleph. In a normalized convention such that the observer always sees himself on $\hat{\pi}^0=\hat{1}$, the big exponential function will always reduce to the regular exponential function if $\Delta\mathbf{x},\Delta t\in\mathbb{R}_0$. This will always be the case for physics confined to $\mathcal{H}$. However, the MCM seeks to expand the realm of physics beyond $\mathcal{H}$ and beyond the local level of aleph. It is hoped that certain quantum effects may be attributed to tunneling or interference effects across levels of aleph. The big exponential function is purposed as a scaffold on which to develop analytical statements of such effects. Other use cases for levels of aleph via the big exponential function include the following.

- All methods for anharmonic potentials in QFT rely on series decompositions of integrals of exponential functions. Decomposition by big rather than little exponential functions may be a useful tool for tackling problems which are currently perceived as intractable.
- The Feynman rules for constructing amplitudes from diagrams might be altered so that a diagram's elements pertain to levels of aleph. In some intuitive way, one would associate QED's enumerated loop corrections with levels of aleph leading to an enhanced understanding of theory.
- Quantum theory's well known perturbative powers series in the fine structure constant may be better interpreted as contributions from different levels of aleph. Each α^n term in a power series would come from the $\hat{\pi}^{\pm n}$ levels measured relative to the observer's location on $\hat{\pi}^0$.
- Levels of aleph were integral to solving the Riemann hypothesis. The architecture [48] of the later direct contradictions [2, 46, 47, 78] was totally reliant on *odd and even levels of aleph* [48]. In the picture described by Figure 6, the even levels of aleph are the coordinate systems attached to $\mathcal{H}_k$. The odd levels refer to another coordinate systems whose origin is in $\varnothing$. The latter would be used to stitch together the even levels, as in Section 1.6.8.
- Though levels of aleph were not cited in computing the characteristic length scale 10^{-4}m (Section 15) [3], the general idea was that contributions from other levels of aleph alter the expected $F_{\text{net}}\hat{z}=\vec{0}$ Newtonian force diagram of a spinning disc in $\mathcal{H}_0$.

1.6.8 A Practical Implementation of Transfinite Numbers

The lack of arithmetic for non-arithmatic numbers makes any parameterization of the unit cell including such numbers inherently cumbersome. Since the observer has no way to measure absolute fractional distance, and since coordinates should always be chosen so as to simplify physics as much as possible, one would seek a parameterization of the path between successive $\mathcal{H}$-branes which does not rely on $x \in \mathbb{F}$. Rather than parameterizing the total extent of χ^4 in one simply connected interval of $\mathbb{R}$ (up to a complex phase), we may use the transfinite continuation $\mathbb{T}$ and a piecewise connected parameter. The transfinitely continued real number line $\mathbb{T}$ follows from the definitions of $\{\mathbb{R}_0, \mathbb{R}_\mathcal{X}, \mathbb{R}_1\}$ extended to the case of $\mathcal{X} > 1$. In the suggested transfinite parameterization, $\varnothing$ lies at $\widehat{\infty}$ relative to an origin in $\mathcal{H}_k$ and sequential $\mathcal{H}$-branes are separated by two levels of aleph. If we assume for simplicity that Ω and $\mathcal{A}$ are colocated at $\varnothing$, $\mathcal{H}_{k+2}$ lies at $2\widehat{\infty}$, etc. The scheme by which one would execute the parameterization as $\chi^4 \in \mathbb{T}$ is outlined in Figures 7-9. The bulk of successive $\Sigma^\pm$ will be doubly charted in coordinates whose origins are in the successive bounding branes. Following an example from real analysis in which the 2-sphere is covered by a double charting of coordinates whose origins are at its two opposite poles, coordinates based in $\mathcal{H}_0$ in the form $\chi^4_{+\{0\}} = \widehat{0} + b$ stretch nearly to Ω.[1] (The subscript $\{k\}$ on $\chi^4_{\pm\{k\}}$ labels the level of aleph.) $\mathcal{F}_0$ is the least upper bound of $\mathbb{R}_0 = \{x \mid \widehat{0} + b \text{ for } |b| < n \in \mathbb{N}\}$ so we say the $\widehat{0} + b$ chart stretches *nearly* to Ω located at $\chi^4 = \mathcal{F}_0$ in the simply connected parameter. Similarly, coordinates measured relative to Ω in the form $\chi^4_{+\{1\}} = \widehat{\infty} - b$ stretch nearly back to $\mathcal{H}_0$. If Ω and $\mathcal{A}$ are colocated, then the coordinates anchored at $\mathcal{A} \cup \Omega \equiv \varnothing$ will also stretch almost to $\mathcal{H}_2$ as $\chi^4_{-\{1\}} = \widehat{\infty} + b$. In terms of $\chi^4_\varnothing$ coordinates, we would write

$$\chi^4_{+\{1\}} \cup \chi^4_{-\{1\}} = \chi^4_{\varnothing\{1\}} \quad . \tag{1.6.23}$$

The $\chi^4_{-\{1\}}$ will overlap with $\chi^4_{-\{2\}} = 2\widehat{\infty} - b$ and so on. The $\chi^4_{\varnothing\{k+1\}}$ on odd levels of aleph will double chart the $\Sigma^\pm$ spanned by $\chi^4_{+\{k\}}$ and $\chi^4_{-\{k+2\}}$. This scheme for double charting between the neighborhood of the origin and the maximal neighborhood infinity makes it possible for us to avoid any reference to the $\mathcal{F}_\mathcal{X}$ numbers for which normal arithmetic is not defined.

Contrary to the lack of arithmetic defined for $x \in \mathbb{F}$, we have already defined a complete system of transfinite arithmetic for $x = n\widehat{\infty}$ when $n \in \mathbb{N}$ [2]. The transfinite continuation should permit a representation of the translation of an observer's reference frame onto a higher level of aleph as nothing but a Galilean transformation (up to a change of scale.) After operating with $\hat{M}^3$ to leave the $\chi^4_{\pm\{0\}} = \widehat{0} \pm b$ coordinates and arrive in the $\chi^4_{\pm\{1\}} = 2\widehat{\infty} \pm b$ coordinates, a coordinate system in the form $\chi^4_{\{1\}} = \widehat{0} \pm b$ is easily recovered by subtracting $2\widehat{\infty}$.[2] Furthermore, this scheme

[1] The hat on $\widehat{0}$ is a convenient notation demonstrating that one may measure distance relative to any origin of coordinates. It is a convention to place zero at the origin but one may measure relative to any other number, such as $\widehat{\infty}$.

[2] Using numbers in the neighborhood of infinity, this section necessarily describes *physical* parameterizations

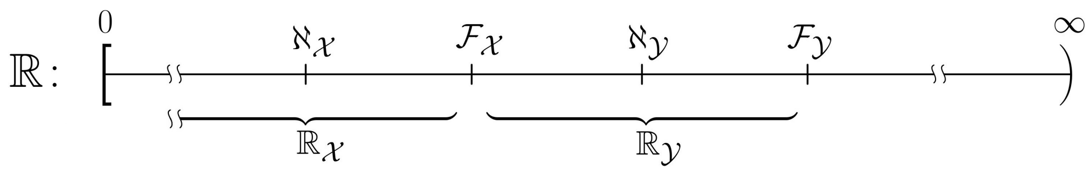

Figure 7: This figure shows the structure of $\mathbb{R}$ as developed in [2]. (The negative branch of $\mathbb{R}$ is omitted.) Due to the lack of standard arithmetic operations for $\mathcal{F}_\mathcal{X} \in \mathbb{F}$, it is desirable that the path between successive $\mathcal{H}$-branes should be parameterized without reference to $\mathcal{F}_\mathcal{X}$.

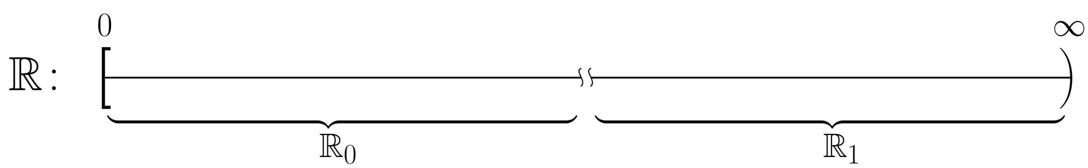

Figure 8: This figure shows the real number line separated between the neighborhood of the origin $\mathbb{R}_0$ and the maximal neighborhood of infinity $\mathbb{R}_1$. Relating to the objects of Figure 7, the neighborhood of the origin $\mathbb{R}_0$ terminates at $\mathcal{F}_0$. Since it is not possible to do arithmetic with non-arithmatic numbers such as $\mathcal{F}_0$ [2], we should introduce some coordinate chart that does not reference them. We propose to introduce a coordinate transformation such that, for instance, every $x = b \in \mathbb{R}_0$ is associated with some $x' = (\widehat{\infty} - b) \in \mathbb{R}_1$, as in Figure 9.

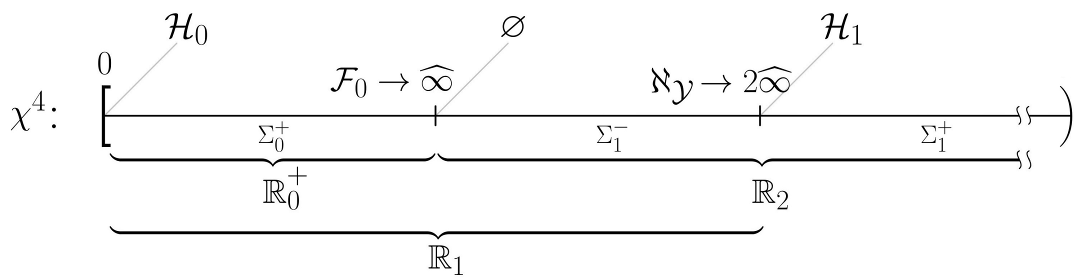

Figure 9: Even levels of aleph are sewn together with odd levels, and vice versa, as in [48]. The midpoint of the least intermediate neighborhood of fractional distance is labeled $\aleph_\mathcal{Y}$. In the scheme where Ω and $\mathcal{A}$ are colocated with $\varnothing$, an intractable $\chi^4 = \mathcal{F}_0$ at the $\Sigma^+ \to \Sigma^-$ step of $\hat{M}^3$ is made tractable by a coordinate transformation in which $\mathcal{F}_0 \to \widehat{\infty}$. Arithmetic, and by extension calculus, is well defined for $\widehat{\infty}$. It is proposed that the 5D bulk of Σ_0^+ should be doubly charted in $\mathbb{R}_0$ and $\mathbb{R}_1$ so that no reference is made to any $x \in \mathbb{F}$ during $\mathcal{H} \to \mathcal{H}$ evolution under $\hat{M}^3$. In this figure's parameterization such that $\chi^4 \in \mathbb{T}$, the non-arithmatic $\mathcal{F}_\mathcal{X}$ are replaced by odd integer multiples of $\widehat{\infty}$. All $\aleph_\mathcal{X}$ are replaced by even integer multiples.

for $\chi^4 \in \mathbb{T}$ restores the original notion of odd and even levels of aleph [1, 48]. To distinguish odd and even levels of aleph, conventions would be amended such that $\varnothing$ is one level higher than $\mathcal{H}_0$ and the forward $\mathcal{H}$-brane is two levels higher. This is intuitive when the coordinates on the $\hat{\pi}^n$ level of aleph are such that $\chi^4_{\{n\}} = n\widehat{\infty} \pm b$. $\chi^4_{+\{1\}} = \widehat{\infty} - b$.

Going *beyond infinity* is not allowed in real analysis but neither is going onto the complex plane and that is standard in physics. Going beyond infinity into $\mathbb{T}$ is only the longitudinal continuation of $\mathbb{R}$ in the way that going onto $\mathbb{C}$ is the transverse continuation. All the tools of complex analysis have the highest utility in physics and we suggest that any tools developed in the transfinite analysis of fractional distance are likely to be equally useful.

1.6.9 Further Considerations for Even and Odd Levels of Aleph

Consider the limit of $\aleph_x$ as x goes to 0. For any $x > 0$, this number has non-vanishing fractional distance and must be greater than any $n \in \mathbb{N}$. From this we conclude

$$\lim_{x \to 0} \aleph_x \neq 0 \quad . \tag{1.6.24}$$

Since $\mathcal{F}_0$ is defined to be the least real number greater than every natural number, a reasonable supposition is

$$\lim_{x \to 0} \aleph_x = \mathcal{F}_0 \quad . \tag{1.6.25}$$

If $\aleph_0 = \mathcal{F}_0$, then every other $\mathcal{F}_\mathcal{X}$ should also be some $\aleph_\mathcal{X}$. Thus, we might suppose that the piecewise double charting suggested in Figure 9 is naturally as in Figure 10. However, the double charting of intervals in two simultaneous neighborhoods of infinity $\mathbb{R}_\mathcal{X} \neq \mathbb{R}_\mathcal{Y}$ is such that

$$x \in \mathbb{R}_\mathcal{X}, \mathbb{R}_\mathcal{Y} \quad \Longrightarrow \quad \frac{x}{\infty} = \mathcal{X} \quad , \quad \text{and} \quad \frac{x}{\infty} = \mathcal{Y} \quad . \tag{1.6.26}$$

To resolve this contradiction, we might assign $\mathbb{R}_\mathcal{Y}$ as an odd level of aleph and say that all *odd* $\{\aleph_\mathcal{Y}\}$ are the immeasurable $\mathcal{F}_\mathcal{X} \in \mathbb{F}$. Despite zero being even, these levels are labeled odd so that we may set the neighborhood of the origin as an even level. The non-arithmatic property would be associated with the separation of $\mathcal{X}$ and $\mathcal{Y}$ by

along χ^4. The abstract coordinates are introduced so that we may describe distances along χ^4 with numbers in the neighborhood of the origin. So, to the extent that we have suggested that infinite relative scale between unit cells should be encoded on the k quantum number in $|\psi; \hat{\pi}^k\rangle$, the Galilean transformation subtracting $2\widehat{\infty}$ might be associated with subtracting 2π in the abstract coordinates. This might be further associated with the 2π in $\hat{M}^3$'s returned value $2\pi\Phi$.

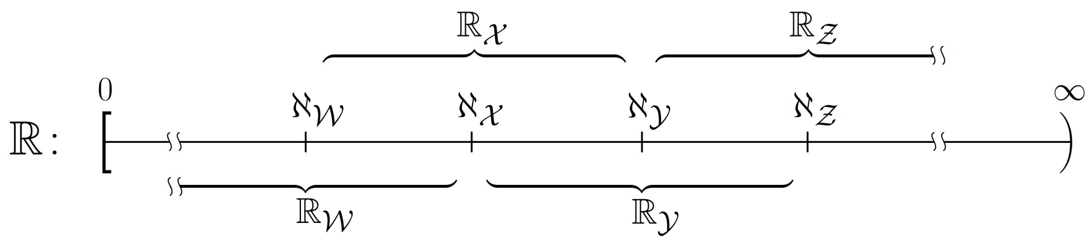

Figure 10: Compare this figure to Figures 7 and 9. In this scheme, odd levels of aleph are associated with immeasurable numbers. The connectedness of $\mathbb{R}$ would require that odd neighborhoods are topologically closed because the even neighborhoods are open.

the least positive real number such that

$$\mathcal{X} - \mathcal{Y} = \text{undefined} \quad . \tag{1.6.27}$$

However, the Cauchy sequences definition of $\mathbb{R}$ might suggest that $\mathcal{X} - \mathcal{Y} = 0$ so further analysis is required. The non-arithmatic odd neighborhoods of fractional distance would be distinguished from the open, even neighborhoods by topological closure. We will not use this convention in the present book. It is mentioned mainly because $\aleph_0 = 0$ was given in the main paper on fractional distance analysis [2] while that equality may not be supported by the ε–δ formalism, and because $\aleph_0 = \mathcal{F}_0$ supports a desirable construction for even and odd levels of aleph.

As a consequence of this scheme for odd and even levels of aleph, one might suppose that the non-definition of $\chi^4_\pm = 0$ is better characterized by the location of the $\mathcal{H}$-brane at a non-arithmatic value of χ^4 in the reversed convention for choosing even and odd.

1.7 Operators, States, and the Schrödinger Equation

1.7.1 $\hat{M}^3$ as a Translation Operator

The usual quantum theory implements time evolution between measurements as diffusion (or oscillation) followed by collapse. The MCM supplements the usual theory of successive measurements in $\mathcal{H}$ with intermediate steps at Ω and $\mathcal{A}$. Therefore, given $\hat{M}^3 = \prod_\lambda \hat{M}_\lambda$ one might take $\hat{M}_\lambda$ as an ordinary translation operator $\hat{\mathcal{J}}_\lambda$ such that for $\lambda \in \{+, -, \varnothing\}$ we would have

$$\hat{M}_\lambda \equiv \hat{\mathcal{J}}_\lambda(\Delta\chi^4_\lambda) = c_\lambda \exp\left\{ -\frac{i\hat{p}_\lambda \Delta\chi^4_\lambda}{\hbar} \right\} \quad , \quad \text{with} \qquad \hat{p}_\lambda = -i\hbar\partial_\lambda \quad . \tag{1.7.1}$$

(See Appendix B for a review of the translation operator $\hat{\mathcal{J}}$.) In this way, $\hat{M}^3$ would send states across the unit cell as

$$\begin{aligned}\hat{M}^3\big|\psi,\hat{\pi}^0\big\rangle &= \hat{\mathcal{J}}_-\,\hat{\mathcal{J}}_\varnothing\,\hat{\mathcal{J}}_+\big|\psi;\hat{\pi}^0\big\rangle \\ &= \pi\,\hat{\mathcal{J}}_-\,\hat{\mathcal{J}}_\varnothing\big|\psi;\hat{\Phi}^0\big\rangle \\ &= \Phi\pi\,\hat{\mathcal{J}}_-\big|\psi;\hat{2}^1\big\rangle \\ &= 2\pi\Phi\big|\psi;\hat{\pi}^1\big\rangle \quad .\end{aligned} \tag{1.7.2}$$

There are a number of problems with this definition for $\hat{M}^3$. These deficiencies provide guidance toward a better analytical representation.

- The unit cell is such that for $\mathcal{H}$ located at $\lim\chi^4_\pm\!\to\!0$, we have $\mathcal{A}$ at $\chi^4_-=-\varphi$ and Ω at $\chi^4_+=\Phi$. This allows us to define appropriate $\hat{\mathcal{J}}$ with $\Delta\chi^4_+=\Phi$ and $\Delta\chi^4_-=\varphi$. (The latter is supplemented by an understanding that $\Delta\chi^4_-$ is defined according to the scale of the forward level of aleph and that it must increase in the opposite direction to χ^4_+.) However, the step $\Omega\to\mathcal{A}$ may be more like a time reversal or reflection than a translation operation. If Ω is a black hole and $\mathcal{A}$ is a white hole connected by a zero distance wormhole (the case in which Ω and $\mathcal{A}$ are colocated at $\varnothing$ rather than bounding a region containing it), a reversal of the time arrow may be all that is needed to execute $\Omega\to\mathcal{A}$. However, it is not yet determined whether $\mathcal{A}$ and Ω bound the region containing $\varnothing$ or if they are colocated there. (These cases are discussed in Section 4.) So, it is not clear that the $\Omega\to\mathcal{A}$ step involves any translation at all. If it does, simple translation cannot tell the whole story because the metric signature changes between $\Sigma^\pm$. Waves (or heatlike solutions) cannot be simply transmitted through the obstruction in the topology induced by the changing metric signature.

- With subscripts running over $\{+,-,\varnothing\}$, one would assume $[\hat{p}_j,\hat{p}_k]=0$ and consequently $[\hat{M}_j,\hat{M}_k]=0$. If these operators commute, then we should be able to reorder them but that is not consistent with the overall idea. For instance, the $\hat{M}_2$ operator executing $\Omega\to\mathcal{A}$ should only act on states in Ω. It may not make sense for it to act on other states.

- $\hat{\mathcal{J}}$ executes equal-time parallel transport. Since observation B necessarily takes place at some chronological time later than that associated with observation A, the translation operator alone is not sufficient to accomplish the task. The state $\hat{M}^3|\psi;\hat{\pi}^n\rangle = c|\psi;\hat{\pi}^{n+1}\rangle$ must show up in $\mathcal{H}_{n+1}$ with a time that agrees with $\hat{\mathcal{U}}(t_{n+1},t_n)|\psi,t_n\rangle = |\psi,t_{n+1}\rangle$. In other words, MCM time evolution must incorporate Schrödinger evolution as a simultaneous process during transit of the unit cell. Static transport by $\hat{M} \propto \hat{\mathcal{J}}$ cannot agree with time-dependent experimental results.

1.7.2 $\hat{M}^3$ as a Ladder Operator

$\hat{M}^3$ is like a ladder operator for the level of aleph. It increases the k quantum number when it operates on $|\psi, \hat{\pi}^k\rangle$. To better understand $\hat{M}^3$ and its associated constant $2\pi\Phi$, we will look at the Dirac ladder operators

$$\hat{a}^\dagger = \sqrt{\frac{m\omega}{2\hbar}}\left(\hat{x} - \frac{i\hat{p}}{m\omega}\right) \quad , \quad \text{and} \qquad \hat{a} = \sqrt{\frac{m\omega}{2\hbar}}\left(\hat{x} + \frac{i\hat{p}}{m\omega}\right) \quad . \tag{1.7.3}$$

They raise and lower the n quantum number for states in the simple harmonic oscillator (SHO) potential. Such states are denoted $|n\rangle$. Dirac notation is such that

$$\hat{a}^\dagger\big|n\big\rangle = \sqrt{n+1}\big|n+1\big\rangle \quad , \quad \text{and} \qquad \hat{a}\big|n\big\rangle = \sqrt{n}\big|n-1\big\rangle \quad . \tag{1.7.4}$$

So far, we have treated $\hat{M}^3$ only in the Dirac notation

$$\hat{M}^3\big|\psi; \hat{\pi}^n\big\rangle = 2\pi\Phi\big|\psi, \hat{\pi}^{n+1}\big\rangle \quad , \tag{1.7.5}$$

without first writing down its analytical expression, as in (1.7.3). Namely, (1.7.4) is only a shorthand developed after Schrödinger's equation was solved for the SHO Hamiltonian

$$\hat{H} = \frac{\hat{p}^{\,2}}{2m} + \frac{m\omega^2\hat{x}^{\,2}}{2} \quad , \quad \text{with} \qquad \omega = \sqrt{\frac{k}{m}} \quad . \tag{1.7.6}$$

The solution is

$$\big|n\big\rangle = \phi_n(x) = \frac{1}{\pi^{1/4}\big(2^n n!\big)^{1/2}} H_n(x)\, e^{-x^2/2} \quad , \tag{1.7.7}$$

where H_n is the n^{th} Hermite polynomial. This result shows that the real physics of (1.7.4) comes from (1.7.7) and (1.7.3). Operation with $\hat{a}$ and $\hat{a}^\dagger$ on $\phi_n(x)$ *provably* yields $\hat{a}^\dagger|n\rangle = \sqrt{n+1}|n+1\rangle$ and $\hat{a}|n\rangle = \sqrt{n}|n-1\rangle$. For $\hat{M}^3$, we have jumped into the end result of the operator algebra $\hat{M}^3|\psi, \hat{\pi}^n\rangle = 2\pi\Phi|\psi; \hat{\pi}^{n+1}\rangle$ without first finding the analytical representation of $\hat{M}^3$. On top of that, we have suggested that a more complicated equation than Schrödinger's equation is needed for $\hat{M}^3$ without writing that equation down and solving for its states, i.e.: SHO states are such that

$$\big|n\big\rangle = \frac{1}{\sqrt{n!}}\big(\hat{a}^\dagger\big)^n\big|0\big\rangle \quad , \tag{1.7.8}$$

but we have not yet found an analytical form for $\hat{M}^3$ with which to *provably* write

$$\big|\psi;\hat{\pi}^n\big\rangle = c_n\big(\hat{M}^3\big)^n\big|\psi;\hat{\pi}^0\big\rangle \quad . \tag{1.7.9}$$

Even if we did have the analytical form of $\hat{M}^3$, all we know about $|\psi;\hat{\pi}^0\rangle$ is that it must reduce to the corresponding quantum mechanical $|\psi\rangle$ in the limit of $\chi^4_\pm \to 0$. That may or may not be a trivial constraint. As SHO states are uniquely determined from the SHO Hamiltonian and Schrödinger's equation jointly, MCM analogues of these important fundamentals are required.

Regarding the discovery of the Schrödinger equation, Schrödinger deduced it (or guessed it) following a process of trial and error [79, 80]. He was well directed in his search by an understanding that the equation for the wavefunction should be first order in its time derivative but the MCM has two kinds of time and the expected equation for $\hat{M}^3$ should be third order in at least one of them. Thus, a potential iterative development process searching for the MCM equation may be far more cumbersome than Schrödinger's search for his eponymous heat equation. Luckily, we will observe in Section 1.11 that certain results suggest a narrowing of the field of all possible equations.

1.7.3 MCM Plane Wave States

Kaluza–Klein theory requires that there should not exist any 5D matter-energy in the bulk of $\Sigma^\pm$. This suggests that we should treat the bulk as free space devoid of any potential energy landscape. The Hamiltonian operator for free space in one dimension is

$$\hat{H}_0 = -\frac{\hbar^2}{2m}\partial_x^2 \quad . \tag{1.7.10}$$

The solutions to the according Schrödinger's equation are plane waves:

$$\phi(x,t) = \exp\big\{i\big[kx-\omega(k)t\big]\big\} \quad , \quad \text{with} \qquad \omega(k) = \frac{\hbar k^2}{2m} \quad . \tag{1.7.11}$$

In the position representation, infinite plane waves are momentum eigenstates. Free momentum eigenstates cannot be observed so, referring to the rigged Hilbert space $\{\mathcal{H}',\mathcal{A}',\Omega'\}$, infinite plane waves cannot live in Hilbert space $\mathcal{A}'$. On the other hand, plane waves in a finite region V are constrained by $\phi'(\partial V) = 0$ where ∂V is the boundary of V. Subject to this boundary condition, ϕ' is normalizable and can belong to $\mathcal{A}'$.

The main utility of infinite plane waves is for the construction of wavepackets which are normalizable and observable, even in unbounded regions:

$$u(x,t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dk\, A(k) \underbrace{\exp\big\{ i \big[kx - \omega(k) t \big] \big\}}_{\phi(x,t)} \quad \Longrightarrow \quad u \in \mathcal{A}' \ . \tag{1.7.12}$$

The infinite plane waves in the integrand are Fourier transforms of Dirac δ functions. Such functions and their Fourier transforms, two representations of the same state, only live in Ω'. So, since plane waves (*i*) satisfy the Schrödinger equation, (*ii*) are the analytical basis for all-important wavepackets, and (*iii*) they appeal to the small sliver of extra freedom afforded by the Ω' part of the MCM's rigged Hilbert space, ϕ is an appropriate state for the presumed energy landscape between two instances of $\mathcal{H}$. The search for an $\hat{M}^3$ equation should start with $\hat{M}^3$ acting on plane waves.

The MCM equation should contain derivatives with respect to χ^4 so the ansatz for an MCM plane wave will be

$$\psi(x,t,\chi^4_\pm) = \exp\big\{ i \big(kx - \omega t \pm \chi^4_\pm \big) \big\} \ , \tag{1.7.13}$$

where $\chi^4_\pm$ implicitly includes a $\chi^4_\varnothing$ case, if needed. (We intermingle physical and abstract coordinates in the ansatz only for simplicity.) Appealing to χ^4 as a non-physical, abstract coordinate, we will assume it is dimensionless and does not require an analogue of k or ω.[1] $\psi(x,t,\chi^4_\pm)$ reduces to the QM wavefunction $\phi(x,t)$ in the limit $\chi^4_\pm \to 0$ corresponding to $\mathcal{H}$. This limiting behavior is a hard constraint on the theory since QM is known to agree with experiment. As per usual in physics, one starts with a boundary condition and develops solutions accordingly. The present boundary condition is that a plane wave in the bulk must reduce to an ordinary plane wave in $\mathcal{H}$.

Landau's treatment of plane waves is demonstrative [81].

> "A plane wave is a mathematical abstraction, a solution to the wave equation which has constant phase along a 2D infinite plane. Although these may not be physically realizable, they are a convenient substitute for a wave packet of definite momentum and are the conventional basis for expanding the wave function of an interacting particle. The wave functions

[1] As plane waves are developed, we will choose to include a coefficient as a scale factor whether or not χ^4 is dimensionless.

of quantum mechanics form a Hilbert space, that is, a linear vector space of infinite dimension. Whereas the dynamical coordinates **r** and **p** of wave functions are continuous, the eigenvalues or parameters of these functions, such as the bound-state energies $E = -\kappa_i^2/2\mu$ are discrete. Any Hermitian Hamiltonian can be used to generate a complete, orthogonal set of wave functions. The free-particle Hamiltonian,

$$H_0 = \frac{\mathbf{p}^2}{2\mu} = -\frac{\boldsymbol{\nabla}}{2\mu} \quad , \tag{1.7.14}$$

is particularly convenient because it generates the plane waves:

$$\begin{aligned}
\tilde{\mathbf{p}}\phi_{\mathbf{k}}(\mathbf{r}) &= \mathbf{k}\phi_{\mathbf{k}}(\mathbf{r}) \ , & k &= |\mathbf{k}| \\
H_0\phi_{\mathbf{k}} &= E_k\phi_{\mathbf{k}}(\mathbf{r}) \quad , & E_k &= k^2/2\mu \\
\phi_{\mathbf{k}}(\mathbf{r}) &= Ne^{i\mathbf{k}\cdot\mathbf{r}} \quad , & N &= \begin{cases} \left(2\pi\right)^{-3/2} & \text{infinite domain} \quad , \\ V^{-1/2} & \text{finite domain} \quad . \end{cases}
\end{aligned} \tag{1.7.15}$$

For simplicity in developing the formalism (and a patina of mathematical rigor), it is useful to consider the plane waves as occupying a finite volume (a box.) The box and the periodic boundary conditions we impose on the wave functions are just for convenience (scattered waves are certainly not periodic); eventually we will go to the limit of an infinite domain.

“**Little Boxes**

“To determine the allowed eigenenergies, we place the plane waves $[\phi_{\mathbf{k}}(\mathbf{r})]$ in a box of volume V with sides (L_x, L_y, L_z), and demand that they satisfy the periodic boundary conditions

$$\begin{aligned}
&\phi_{\mathbf{k}}(x + L_x, y + L_y, z + L_z) = \phi_{\mathbf{k}}(x, y, z) \quad , \\
&\Longrightarrow \qquad (k_x L_x, k_y L_y, k_z L_z) = 2\pi(i_x, i_y, i_z) \quad .
\end{aligned} \tag{1.7.16}$$

Here $(i_x, i_y, i_z) \equiv i$ is a set of three positive or negative integers which determine the allowed, discrete wave vectors and thus energies:

$$\mathbf{k}_i = 2\pi\left(\frac{i_x}{L_x}, \frac{i_y}{L_y}, \frac{i_z}{L_z}\right) \quad , \qquad E_i = \frac{k_i^2}{2\mu} \quad . \tag{1.7.17}$$

With these boundary conditions, the plane waves for different values of i and j are *orthogonal*. By choosing the normalization constant N we make the plane waves *orthonormal*:

$$\begin{aligned} \phi_{\mathbf{k}_i}(\mathbf{r}) \equiv \phi_i(\mathbf{r}) &= \frac{e^{i\mathbf{k}_i\cdot\mathbf{r}}}{\sqrt{V}} \quad , \\ \implies \qquad \int d^3r \phi_i^*(\mathbf{r})\phi_j(\mathbf{r}) &= \delta_{ij} \quad , \qquad \text{(orthonormality)} \quad . \end{aligned} \tag{1.7.18}$$

Note that in the confined volume of the box, the variable $\mathbf{k}$ is discrete but the variable $\mathbf{r}$ is continuous (but limited). The discreteness of $\mathbf{k}_i$ leads to the *Kronecker delta function* in [*(1.7.18)*]. Since the free Hamiltonian is Hermitian, plane waves form a *complete set* in which any solution $\psi(\mathbf{r})$ of Schrödinger's equation can be expanded:

$$\psi(\mathbf{r}) = \sum_i^\infty c_i\phi_i(\mathbf{r}) \quad . \tag{1.7.19}$$

Orthonormality determines the c_i's (multiply [*(1.7.19)*] by ϕ^* and integrate over $\mathbf{r}$):

$$c_i = \int d^3r'\, \phi_i^*(\mathbf{r}')\psi(\mathbf{r}') \quad . \tag{1.7.20}$$

If we substitute this back into [*(1.7.19)*] and interchange the order of integration and summation, we obtain

$$\psi(\mathbf{r}) = \int d^3r' \left[\sum_i^\infty \phi_i^*(\mathbf{r}')\phi_i(\mathbf{r})\right] \psi(\mathbf{r}') \quad . \tag{1.7.21}$$

Yet because [*(1.7.21)*] must be an identity, we identify the term in the brackets as some kind of unit operator. This yields the *closure or completeness relation* for discrete states:

$$\sum_i^\infty \phi_i^*(\mathbf{r}')\phi_i(\mathbf{r}) = \delta(\mathbf{r}' - \mathbf{r}) \quad , \quad \text{(closure)} \quad . \tag{1.7.22}$$

"**The Big Box**

"To obtain plane waves in an infinite domain, we let the box size approach infinity. In this limit of very large L and very large i, the index i is still an integer so $\boldsymbol{\Delta} i \equiv 1$.[1] The momenta $\mathbf{k}_i$ in [*(1.7.17)*] remain finite but

[1]This notation means that the discrete version of the differential element of i, $\boldsymbol{\Delta} i$, is equal to one because that is the smallest increment of change for an integer-valued quantity.

become continuous:

$$\begin{aligned}\frac{2\pi}{L_i}\boldsymbol{\Delta}i \quad &\to \quad d\mathbf{k}_i \ , \quad \boldsymbol{\Delta}i_x \to \frac{L_x}{2\pi}d\mathbf{k}_i \\ \sum\boldsymbol{\Delta}i \quad &\to \quad V\int\frac{d^3\mathbf{k}}{\big(2\pi\big)^3} \quad .\end{aligned} \tag{1.7.23}$$

"[*sic*] To generalize the closure relation [*(1.7.22)*] to a big box, we insert a $\boldsymbol{\Delta}i=1$ into the sum in [*(1.7.22)*], and take the $L\to\infty$ limit:

$$\begin{aligned}&\sum_i^\infty\boldsymbol{\Delta}i\phi_i^*(\mathbf{r}')\phi_i(\mathbf{r})=\delta(\mathbf{r}'-\mathbf{r}) \quad , \\ \implies \quad &V\int\frac{d^3k}{\big(2\pi\big)^3}\frac{e^{-i\mathbf{k}\cdot\mathbf{r}'}}{\sqrt{V}}\frac{e^{i\mathbf{k}\cdot\mathbf{r}}}{\sqrt{V}}=\delta(\mathbf{r}'-\mathbf{r}) \quad , \qquad \text{(closure)} \quad .^1\end{aligned} \tag{1.7.24}$$

This gives the form for plane waves in an infinite domain:

$$\phi_i(\mathbf{r})=\frac{e^{i\mathbf{k}_i\cdot\mathbf{r}}}{\sqrt{V}} \qquad \implies \qquad \phi_\mathbf{k}(\mathbf{r})=\frac{e^{i\mathbf{k}\cdot\mathbf{r}}}{\big(2\pi\big)^{3/2}} \quad . \tag{1.7.25}$$

The orthogonality relation [*(1.7.18)*] for an infinite domain is now just the closure relation with a change of variable,

$$\delta_{ij}\to\int\frac{d^3r}{\big(2\pi\big)^3}\,e^{-i\mathbf{k}'\cdot\mathbf{r}}e^{i\mathbf{k}\cdot\mathbf{r}}=\delta(\mathbf{k}'-\mathbf{k}) \quad \text{(orthogonality)} \quad ." \tag{1.7.26}$$

For disambiguation with the imaginary number i, we will replace Landau's integer i with j in the following. The factor of $(2\pi)^{-3/2}$ in (1.7.25) reflects the fact that time-independent plane waves in an infinite domain are non-physical and cannot be normalized in $\mathbb{R}$. Instead, these states are normalized to the 3D Dirac δ function, as in (1.7.26). Since it is desired that the physical distance between branes exceeds any number in the neighborhood of the origin, the continuous $\mathbf{k}$, unbounded big box case proportional to $(2\pi)^{-3/2}$ should be associated with the physical coordinates. The big box case also describes unbounded plane waves in $\mathcal{H}$ when we take the $e^{i(kx-\omega t\pm\chi^4_\pm)}$ ansatz with $\chi^4=0$. The discrete $\mathbf{k}$, small box case proportional to $V^{-1/2}$ should pertain to the abstract coordinates. The convention in which $\mathcal{A}$ and Ω are surfaces of constant $\chi^4_-=-\varphi$ and $\chi^4_+=\Phi$ is such that $\Sigma^\pm$ are small boxes in the fifth direction.

Consider the orthonormalism of discrete momentum states, as in (1.7.18). The

[1] The Dirac δ function has inverse units to its argument: $\delta(\mathbf{r})$ has units of $[\text{m}^{-3}]$.

orthogonality of ϕ_{j_1} and ϕ_{j_2} when $j_1 \neq j_2$ is well suited to the orthogonality of wavefunctions on different levels of aleph. It was suggested in Section 1.2.5 that the ontological basis might act as lattice vectors for a cosmological lattice in which each lattice site has its own level of aleph specified by some tuple of integers. In that picture, small box plane waves are such that states at different lattice sites are orthogonal. Lattice sites specified by integer combinations of lattice vectors $\{\hat{2}, \hat{\pi}, \hat{\Phi}, \hat{\imath}\}$ are specified with $j \equiv (j_2, j_\pi, j_\Phi, j_i)$ analogous to Landau's $i \equiv (i_x, i_y, i_z)$. One caveat, however, is that the unit cell only requires the small box condition for the $\chi^4_\pm$ directions. It is not yet determined whether $\Sigma^\pm$ should be bounded in the abstract $\chi^\mu_\pm$ coordinates. One is advised that the big or small box convention will depend on the choice of coordinates, and we still have not determined if $\Sigma^\pm$ are bounded in the $\chi^\mu_\pm$ directions (or if $x^4_\pm$ coordinates should exist at all.)

$\{\phi_j\}$ are a complete orthonormal set but the non-unitarity of the $\hat{M}^3$ and/or $\hat{\mathsf{O}}_{\hat{e}_\mu \to \hat{e}_\nu}$ operators suggest that the MCM plane wave basis $\{\psi_j\}$ ought to be orthogonal and *not* orthonormal. The lack of normalism follows from the relative scale between levels of aleph. If the relative scale between the $\mathbb{R}_\mathcal{X}$ and $\mathbb{R}_\mathcal{Y}$ levels of aleph is $\mathcal{C}$, then $\text{len}(\mathbb{R}_\mathcal{X})/\text{len}(\mathbb{R}_\mathcal{Y}) = \mathcal{C}$. In the rescaling $\mathbf{r} \to \mathcal{C}\mathbf{r}$, the 3D wave vector and energy rescale as

$$\mathbf{k}'_j = 2\pi \left(\frac{j_x}{\mathcal{C}L_x}, \frac{j_y}{\mathcal{C}L_y}, \frac{j_z}{\mathcal{C}L_z} \right) \quad , \quad \text{and} \qquad E'_j = \frac{\hbar^2 \mathbf{k}'^2_j}{2m} = \frac{\hbar^2 \mathbf{k}^2_j}{2m\mathcal{C}^2} = \frac{E_j}{\mathcal{C}^2} \quad .^1 \tag{1.7.27}$$

Thus, the energy changes from one lattice site to another. Noting that $\hbar$ has units of $[kg][m^2][s^{-1}]$, the normalization of the observer's reference frame onto the level of aleph where the relative scale is $\mathcal{C}$ may require that meters are redefined to absorb the two factors of $\mathcal{C}$ appearing in the energy's denominator. Presuming $\mathcal{C} \geq 1$, as is the case for $\mathcal{C} = 2\pi\Phi$, the energy decreases with increasing j. This is a positive result because the physical arrow of time never points towards increasing energy in the absence of work. This energy variation may have further applications to the MCM mechanism for dark energy discussed in Section 7. Cosmological redshift is such that photons lose energy with time.

In the preceding, we have considered only some generalized χ^4 without appealing to opposite sign and/or imaginary phase between $\chi^4_\pm$. The behavior of quantum states with real and imaginary wavenumbers is known from

$$k = \frac{\sqrt{2m\big(E - V\big)}}{\hbar} \quad .^2 \tag{1.7.28}$$

[1]Compare to (1.7.17).

The wavenumber k is real when $E > V$. It is imaginary when $E < V$. Coupled with the i in e^{ikx}, we have wave propagation in the classically allowed region where $E > V$ and exponential damping in the classically forbidden region where $E < V$. It was suggested earlier that allowing $\chi^4_\pm$ to be complex will allow us to avoid the metric signature discrepancy at the $\Omega \to \mathcal{A}$ step of $\hat{M}^3$. Now we will suggest an implementation by adding a wavenumber or frequency multiplier to the ansatz as

$$\psi(x,t,\chi^4) = \exp\big\{ i\big(kx - \omega t + \kappa\chi^4\big)\big\} \quad , \quad \text{where} \qquad \kappa = \frac{\sqrt{2m\big(E-V\big)}}{\hbar} \quad . \tag{1.7.29}$$

By choosing an appropriate energy scale on the forward level of aleph, namely $V \in \Sigma^-_{\{1\}}$ higher than $E \in \Sigma^+_{\{0\}}$, we might make the region of metric discrepancy a classically forbidden region so that κ becomes imaginary. Then we will obtain exponential damping of the wavefunction in the region of metric discrepancy:

$$\psi(x,t,\chi^4) = \exp\big\{ i\big(kx - \omega t + i|\kappa|\chi^4\big)\big\} = \exp\big\{ i\big(kx - \omega t\big)\big\} e^{-|\kappa|\chi^4} \tag{1.7.30}$$

Naturally, we have come to a likely resolution for a metric discrepancy between $\Sigma^\pm$ through our consideration of plane wave states. Since we would want damping to increase with penetration into Σ^-, this χ^4 has its origin in $\mathcal{A}$ (or in $\varnothing$ if $\mathcal{A}$ is colocated with Ω). Thus, the energy landscape would steer propagating waves in Σ^+ onto $i\chi^4_-$ spanning another instance of Σ^- where the convention for real and imaginary $\chi^4_\pm$ is reversed (using the freedom to write the signature as either of $\{\mp \pm \pm \pm\}$). This will reduce the topological discontinuities from appearing at $\mathcal{H}$ and $\varnothing$ to $\mathcal{H}$ alone.[1] Everything is reset at $\mathcal{H}$ so there is not so pressing a question of how solutions might be transmitted through it. The act of observation associated with $\mathcal{H}$ gives us more options for dealing with discontinuity there.

Another issue is that the we have associated the region of metric discrepancy with the classically forbidden region of an elementary QM barrier problem but the forbidden region always has the same metric as the allowed region in such problems. Investigation is required to determine whether the usual mechanics of real and imaginary wavenumbers are permitted simultaneously with a changing signature. If the unit cell is constructible so as to avoid a discrepancy at $\varnothing$, then what appears as damping in a 1D QM scattering problem will be manifested in the unit cell as oscil-

[2] This formula for the wavenumber k is standard in elementary QM problems. See Section 2.6 in [82] or Section 2.4 in [83], for example.

[1] In Section 0.2, we introduced a convention in which the 4D metrics in $\Sigma^\pm$ were oppositely signed as $\{\mp \pm \pm \pm\}$. Here, we use the same sign convention $\{-+++\}$ for both sides of the unit cell and add the sign conjugated convention in the spaces crossed by $i\chi^4_\pm$.

lating propagation in the direction perpendicular to the page. In this way, the energy landscape guides undamped propagation in the lattice. If the branch of χ^4 containing the metric discrepancy is classically forbidden, then states will want to avoid it without any need to introduce supplemental mechanisms. The energy landscape will automatically favor continuation on the classically allowed branch.[1] Such conditions are the heart of physics. In the previous sections, we have mostly proposed abstract mathematical mechanisms for what $\hat{M}^3$ is or does. Now we have taken a step toward the physical nitty gritty.

To finish this section, we will mention that a topological mismatch between $\Sigma^\pm$ forbids perfect transmission from one box into another though this is the boundary condition supposed in (1.7.16) if the box is the full unit cell. Barring the obvious case where $\Sigma^\pm$ are two different boxes, one resolution is that we might consider the unit cell as a small but non-trivial box with symplectic geometry between its piecewise $\Sigma^\pm$ parts. Symplectic geometry equips a manifold with a 2-form whose property $dx\wedge dy=-dy\wedge dx$ at least approximates what is intended for the conjugation algebra of $\mathbb{C}^*$ with $(\hat{\varphi}^*)^*\neq\hat{\varphi}$.

1.7.4 The Schrödinger Equation and its Potential Modifications

The Schrödinger equation

$$i\hbar\partial_0\big|\psi,t\big\rangle=\hat{H}\big|\psi,t\big\rangle=\bigg(-\frac{\hbar^2}{2m}\nabla^2+\hat{V}\bigg)\big|\psi,t\big\rangle\quad, \tag{1.7.31}$$

provides an excellent template for what a physical equation looks like. The appearance of both time and space derivatives remedies the problem of equal-time parallel transport cited for $\hat{M}^3$ as a translation operator in Section 1.7.1. With an equation for $\hat{M}^3$, we would obtain its analytical form as we have obtained the ladder operators in Section 1.7.2.

While Schrödinger's equation incorporates the requisite elements of physics lacking in the current description of $\hat{M}^3$, it may or may not be sufficient for MCM evolution on its own. If it is, $\hat{M}^3$ will show up as a new energy in $\hat{H}$:

$$\hat{M}^3\big|\psi;\hat{\pi}^0\big\rangle=2\pi\Phi\big|\psi;\hat{\pi}^1\big\rangle\qquad\longleftrightarrow\qquad\hat{H}_{\text{MCM}}\big|\psi_E\big\rangle=E_{\text{MCM}}\big|\psi_E\big\rangle\quad. \tag{1.7.32}$$

To evaluate this form for $\hat{M}^3$, we must first examine whether or not $|\psi,\hat{\pi}^0\rangle$ is an eigenstate of $\hat{M}^3$. Since $[\hat{\mathcal{U}},\hat{H}]=0$, an energy eigenstate $|\psi_E;t_0\rangle$ is an eigenstate of $\hat{\mathcal{U}}$

[1] If a right-moving wave avoids a forbidden region by diverting onto the directions into *and* out of the page, one might expect attenuation in the lattice. The non-unitary property of $\hat{M}^3$ should counteract this potential for attenuation.

despite the values in the ket changing:

$$\hat{\mathcal{U}}\big|\psi,t_0\big\rangle=\big|\psi;t\big\rangle\quad . \tag{1.7.33}$$

The time dependence boils down to a phase and the state remains the same. Since we have not found the analytical form of $\hat{M}^3$ needed to test whether it commutes with $\hat{H}$, we cannot say if ψ_E is an eigenstate of $\hat{M}^3$. Non-unitarity and changing scale across levels of aleph suggest it may not be. However, the mathematical expression for being sent to a higher level of aleph may be as simple as an accrued $\hat{\pi}$ so that $\hat{\pi}^k\to\hat{\pi}^{k+1}$ in the way that energy eigenstates acquire a phase under operation with $\hat{\mathcal{U}}$: $e^0\to e^{iEt/\hbar}$. If ψ_E is not an eigenstate of $\hat{M}^3$ and $[\hat{M}^3,\hat{H}]\neq 0$, a likely resolution is that $\hat{M}^3$ should satisfy a modified Schrödinger equation. In that case, $\hat{M}^3$ will show up in the time derivative part of an equation which reduces to Schrödinger's equation in the limit of vanishing χ^4 and vanishing derivatives with respect to χ^4. For example, one would consider equations roughly in the form

$$\big(\hat{M}^3+i\hbar\partial_0\big)\big|\psi,t;\hat{\pi}^0\big\rangle=\hat{H}_{\text{MCM}}\big|\psi',t';\hat{\pi}^1\big\rangle\quad . \tag{1.7.34}$$

where $\hat{M}^3$ contains a new time derivative on the left and $\hat{H}_{\text{MCM}}$ contains a new energy on the right.

The unitary time evolution operator $\hat{\mathcal{U}}$ satisfies Schrödinger's equation on its own. We may factor out the $|\psi,t_0\rangle$ time-independent part of $|\psi,t\rangle=\hat{\mathcal{U}}(t,t_0)|\psi,t_0\rangle$ to write an equation for $\hat{\mathcal{U}}$ rather than ψ. In that way, $\hat{M}^3$ may satisfy a time evolution equation without ψ in it at all. This was more or less the original idea in supposing $\hat{\Upsilon}=\hat{\mathcal{U}}+\hat{M}^3$ [3,30]. Given

$$i\hbar\partial_t\hat{\mathcal{U}}=\hat{H}\hat{\mathcal{U}}\quad , \tag{1.7.35}$$

we would write

$$i\hbar\partial_t\hat{\Upsilon}=\hat{H}_{\text{MCM}}\hat{\Upsilon} \tag{1.7.36}$$

or we would seek new equations. We will treat $\hat{\Upsilon}$ in Section 1.11 where its cases for use in an MCM total evolution equation are discussed beyond the modifications presented here.

The remainder of this section catalogs avenues along which Schrödinger's existing equation might be modified without starting over from scratch. This should be useful and/or demonstrative because any new MCM equation should contain Schrödinger's equation as a limit. Possible modifications are listed and then described.

- Schrödinger evolution in χ^4:

$$\partial_0\to\partial_4 \tag{1.7.37}$$

- A time gradient:

$$\partial_0 \to \widetilde{\nabla} = \partial_0 \mathbb{1} + \partial_4 \hat{\Phi} \quad , \quad \text{where} \qquad (\mathbb{1}, \hat{\Phi}) = (\hat{\pi}^0, \hat{\Phi}^1) \tag{1.7.38}$$

- Momentum in the χ^4 direction:

$$\nabla_i^2 \to \widehat{\nabla}^2 = \nabla_i^2 + \nabla_4^2 \tag{1.7.39}$$

- A separable potential energy:

$$\hat{H} \to \hat{H} = \hat{H}_0 + \hat{V}(x,t) + \hat{V}_{\text{MCM}}(\chi^4, t) \tag{1.7.40}$$

- A non-separable potential energy:

$$\hat{H} \to \hat{H} = \hat{H}_0 + \hat{V}_{\text{MCM}}(x, \chi^4, t) \tag{1.7.41}$$

- Classical transport of a quantum system:

$$\dot{\psi} \to x \qquad \Longrightarrow \qquad F_{\text{net}} = m\ddot{x} \to m\dddot{\psi} \tag{1.7.42}$$

Schrödinger Evolution in $\boldsymbol{\chi^4}$ An elementary modification $\partial_0 \to \partial_4$ on the left side of Schrödinger's equation is such that

$$i\hbar \partial_4 \big|\psi, \chi^4\big\rangle = \hat{H}\big|\psi, \chi^4\big\rangle \quad . \tag{1.7.43}$$

This equation is well suited to a further resolution of χ^4 into its piecewise parts: χ^4_+, $\chi^4_\varnothing$, and χ^4_-. It was suggested in [84] that the steps of $\mathcal{H} \to \Omega \to \mathcal{A} \to \mathcal{H}$ might be motions derived from three integrated Schrödinger equations using ∂_+, $\partial_\varnothing$, and ∂_- in place of ∂_t on the LHS. Such a description by concatenated integration paths necessarily relies on the sum of three operations rather than the product $\prod \hat{M}_i$ which has been supposed. This might be resolved by moving the $\hat{M}_i$ into an exponential function such that

$$\hat{M}^3 = \prod_{k=1}^{3} e^{\hat{M}_k} \quad . \tag{1.7.44}$$

This form is familiar from the $\hat{\mathcal{U}} = e^{-i\hat{H}t/\hbar}$ chronological time evolution operator which $\hat{M}^3$ complements as the chirological evolution operator. Exponential structure in $\hat{\mathcal{U}}$

underpins the path integral as

$$\begin{aligned}\big\langle x_{\rm I}\big|e^{-i\hat{H}t/\hbar}\big|x_{\rm F}\big\rangle &= \big\langle x_0\big|e^{-i\hat{H}\delta t/\hbar}e^{-i\hat{H}\delta t/\hbar}...e^{-i\hat{H}\delta t/\hbar}\big|x_N\big\rangle \\ &= \left(\prod_{k=1}^{N-1}\int dx_n\right)\big\langle x_0\big|e^{-i\hat{H}\delta t/\hbar}\big|x_1\big\rangle\big\langle x_1\big|e^{-i\hat{H}\delta t/\hbar}\big|x_2\big\rangle\big\langle x_2\big|...\big|x_N\big\rangle \ ,\end{aligned} \tag{1.7.45}$$

so (1.7.44) is well suited to piecewise evolutions along MCM cosmological lattice vectors. Taking $\chi^4_\varnothing$ to have no linear extent, meaning the case in which Ω and $\mathcal{A}$ are colocated at $\varnothing$, one might substitute the requisite chronological evolution $|\psi,t_0\rangle\to|\psi,t_1\rangle$ for the $\partial_\varnothing$ step of $\hat{M}^3$. There is some likeness between $\mathcal{H}$ and $\varnothing$ as obstructions between $\Sigma^\pm$ but the mechanism by which we might associate t and $\chi^4_\varnothing$ remains to be investigated. Furthermore, the dimensions of (1.7.43) are contrary to the previous convention in which χ^4 is dimensionless.

A Time Gradient The time gradient $\widetilde{\nabla}$ follows from $\partial_0\!\to\!\partial_4$. Rather than replacing ∂_0 with ∂_4, we supplement the Schrödinger equation's chronological time derivative with an added chirological part:

$$i\hbar\widetilde{\nabla}\big|\psi\big\rangle = i\hbar\big(\partial_0\hat{\pi}+\partial_4\hat{\Phi}\big)\big|\psi\big\rangle = \hat{H}\big|\psi'\big\rangle \ \ . \tag{1.7.46}$$

A deficiency is that the gradient ought to include components for the other ontological basis vectors as

$$\widetilde{\nabla} = \partial_0\hat{\pi}+\partial_+\hat{\Phi}+\partial_\varnothing\hat{\imath}+\partial_-\hat{2} \ \ , \tag{1.7.47}$$

but this does not appear to respect the ordering of the $\mathcal{H}\!\to\!\Omega\!\to\!\mathcal{A}\!\to\!\mathcal{H}$ steps. That might be remedied if the disordered derivatives vanish as needed during piecewise motions across the unit cell. Perhaps $\hat{2}$ and $\hat{\imath}$ should be removed from the time gradient on the grounds that they indicate physical and abstract space as we have used $\hat{\pi}$ and $\hat{\Phi}$ to indicate physical and abstract time.

As written in (1.7.46), integrated motion along χ^4 would raise the level of aleph with $\hat{\Phi}$ acting on the χ^4 part of $|\psi,t,\chi^4\rangle$ but the x^0 part does not raise it with $\hat{\pi}^0=\mathbb{1}$. Operation with the time gradient yields wavefunctions on two levels of aleph. Following the plane wave prescription in the previous section, wavefunctions on different levels of aleph are orthogonal. Hence, $\hat{H}$ operating on ψ would have to result in the sum of two orthogonal states. This is not the behavior usually associated with the $\hat{H}$ operator.

Momentum in the χ^4 Direction Canonical quantization in the position representation is such that

$$p_i \to -i\hbar\partial_i \quad . \tag{1.7.48}$$

One would assume that momentum in the χ^4 direction quantizes as

$$p_4 \to -i\hbar\partial_4 \quad . \tag{1.7.49}$$

The kinetic part of the Hamiltonian would be altered as

$$\hat{H}_0 = -\frac{\hbar^2}{2m}\widehat{\nabla}^2 = -\frac{\hbar^2}{2m}\sum_{k=1}^{4}\partial_k^2 \quad . \tag{1.7.50}$$

Within ∂_4, the $\{\chi^4_+, \chi^4_\varnothing, \chi^4_-\}$ structure is such that each variant should be given its own derivative. In the picture of ontological basis vectors as cosmological lattice vectors, one would assume the possibility for arbitrary momenta in the form $\mathbf{p}_4 = (a\hat{\chi}^4_+, b\hat{\chi}^4_\varnothing, c\hat{\chi}^4_-)$. In that case, one might omit the spatial momentum of physical 3-space to write

$$\hat{H}'_0|\psi\rangle = -\frac{\hbar^2}{2m}\big(\partial_+^2 + \partial_\varnothing^2 + \partial_-^2\big)|\psi\rangle \quad . \tag{1.7.51}$$

However, (1.7.51) assigns physical dimension to the abstract coordinates which probably ought to be dimensionless. In that case, one would drop the $\hbar^2/m$ from (1.7.51) with an intention to write a Schrödinger equation completely in the abstract coordinates. Furthermore, (1.7.49) may not be the correct quantization prescription at all. The three-fold structure on χ^4 is such that its quantization prescription might be exotic.

A New Separable or Non-Separable Potential Energy Function While the KKT requirement for a vanishing 5D Ricci tensor is an obstacle to the direct introduction of a new polynomial energy function of χ^4, the physical concept of a unit cell invokes a regular, periodic potential energy function. Such a function is the foundation of lattice physics. An upside down Dirac comb forbidding the bulk of $\Sigma^\pm$ while allowing the labeled branes seems like an energy that would motivate $\mathcal{H}\to\Omega\to\mathcal{A}\to\mathcal{H}$ as a generalized Euler–Lagrange process. What a new periodic term in $\hat{H}$ might be when KKT requires no 5D matter-energy deserves further study.

Another issue is that we have no units for χ^4 (yet) but any new energy function must be quantified in Joules if it is of the separable variety. Non-constant energy functions always depend on the units of the coordinates to achieve the dimensionality of Joules. An example of a non-separable new energy function not requiring dimensionful χ^4 is one where a dimensionless piece associated with the unit cell multiplies part (or all) of an existing Hamiltonian. This would represent, for example, the scale factor for changing energies across changing levels of aleph (Section 1.7.3). For dimensionful χ^4, the MCM plane wave ansatz must be revised as

$$\psi(\mathbf{x},t,\chi^4_\pm) = \exp\big\{i\big(\mathbf{k}\cdot\mathbf{x} - \omega t + \beta_\pm \chi^4_\pm\big)\big\} \quad , \tag{1.7.52}$$

where $\beta_\pm$ is a frequency or wavenumber analogue. If we are to keep the Schrödinger equation's time derivative part as it is, the only possibility for new physics is a new energy term. While this strongly suggests that a fundamental modification to the time derivative part of Schrödinger's equation is required, we will briefly examine the case in which a third derivative associated with $\hat{M}^3$ appears as a new energy term.

Operating on ψ with ∂_4^3 will bring down three powers of the scalar β. As written, (1.7.52) allows plane waves to propagate only along cosmological lattice vectors. To add propagation in the direction of arbitrary superpositions of lattice vectors, which is to allow waves with arbitrary $\mathbf{p}_4 = (p_+, p_\varnothing, p_-)$, the ansatz must be revised as

$$\psi(\mathbf{x},t,\boldsymbol{\chi}^4) = \exp\big\{i\big(\mathbf{k}\cdot\mathbf{x} - \omega t + \boldsymbol{\beta}\cdot\boldsymbol{\chi}^4\big)\big\} \quad , \quad \text{with} \qquad \boldsymbol{\chi}^4 = (\chi^4_+, \chi^4_\varnothing, \chi^4_-) \quad . \tag{1.7.53}$$

In this case, the third derivative will bring down a vector $|\boldsymbol{\beta}|^2\boldsymbol{\beta}$ resulting in an eccentric analytical expression:

$$i\hbar\partial_0\psi = \big(\hat{H}_0 + i\partial_4^3\big)\psi = \left(\frac{|\mathbf{p}|^2}{2m} + |\boldsymbol{\beta}|^2\boldsymbol{\beta}\right)\psi \quad . \tag{1.7.54}$$

What would be the meaning of the sum of a scalar and a vector? The main venue for such a sum in physics is the quaternions. The sum of a vector and a scalar cannot be written off immediately as nonsensical because the MCM Hamiltonian for time arrow spinors (Section 12) is quaternion-valued [84]. Furthermore, the behavior of even derivatives to return scalars and odd derivatives to return vectors may be useful in a scheme for separating odd and even levels of aleph.

Classical Transport of a Quantum System Since the introduction of a third derivative into the formalism is desired, we might combine the first order Schrödinger equation with Newton's second order force law such that

$$i\hbar\dot{\psi} = \hat{H}\psi \quad , \quad \text{and} \qquad m\dddot{\psi} = F_{\text{MCM}} \quad . \tag{1.7.55}$$

The time derivative of ψ replaces the classical position x. This supplementation of

Schrödinger's equation as a classical trajectory for $\dot{\psi}$ across the unit cell may be useful for avoiding KKT Ricci tensor violations in the bulk because the quantity $\dot{\psi}$ is not directly associated with matter-energy distributions. It is only the rate of change of a complex-valued probability amplitude.

As an off-the-cuff example of what is meant by classical transport of a quantum system, consider that lattice physics is an extended application of Hooke's law. Restricted to positive displacements, Hooke's law is

$$m\ddot{x} = kx \quad \Longrightarrow \quad \dddot{x} = \frac{k}{m}\dot{x} \ . \tag{1.7.56}$$

One might attempt to associate the oscillation of masses connected by springs (lattice sites) with the the oscillation of the wavefunctions attached to each lattice site. Since the Hamiltonian is constructed from the the Lagrangian as $H = \sum p\dot{q} - L(q, \dot{q})$, (1.7.56) offers an easy way to introduce a third derivative term into the energy function. Substituting the oscillation of position with the oscillation of the wavefunction allows us to put the third derivative directly into the $L(q, \dot{q})$ function with $\dddot{x} \propto \dot{x}$.

1.8 Wavefunction Collapse

1.8.1 A Possibility for Retrocausality

A good and modern overview of issues related to retrocausality in wavefunction collapse is found in [85]. To paraphrase briefly, Ellerman's thesis is that Schrödinger's cat is in an entangled superposition of life and death eigenstates while the box is closed, and that opening the box does not retrocausally affect the life or death of the cat during that time. Rather, opening the box forces the collapse of the life/death superposition into one eigenstate or the other by placing a detector outside of the box. Detectors are modeled in QM as operators which project quantum systems onto their eigenstates. If the box is opened at time t, then the wavefunction is collapsed only for times later than t. Ellerman contends, rightly, that the language of QM is not such that we may determine the life or death of the cat prior to the measurement. This writer's minor criticism, however, is the lack of a caveat: Ellerman assumes that QM is the correct description of nature. He discounts the possibility that QM is merely a hack allowing us to predict experiments' results. He does not contextualize the possibility that such effects as retrocausality may be objectively *real* even while QM does not predict them. What is real or not is a matter of semantics, *or not*, but it remains true that there may exist a better description of reality than QM. The interpretation of that other description might suggest retrocausality.

Even while this writer agrees with Ellerman regarding the interpretation of QM, it is not known what is inside the closed box. Not knowing what is inside is different that knowing that there is a superposition. If QM's usual interpretation is correct, which we have fair reason to suspect, then we would know that the cat exists as a superposition until a detector projects it into one of its life eigenstates. Still, the reader is encouraged to understand that opening the box may, in fact, retrocausally affect the life or death of the cat because ignorance of the cat's state is not exactly knowledge that the state is a superposition. That implication depends on an assumption that QM is more than just a hack for telling the results of experiments. Obviously, this writer's opinion is that QM is exactly that. There probably does exists a better description than QM. Whether or not a better theory would preclude retrocausality is unknown. The context of retrocausality in the MCM is that the EM potential A^μ in $\mathcal{H}$ is a superposition of contributions from $A^\mu_\pm$ in $\Sigma^\pm$ (Section 16) so it follows that physics in the present is at least retrocausal from the abstract future $\chi^4_+ > 0$. How that may or may not relate to objective chronological retrocausality from the Minkowskian future light cone remains to be determined.

1.8.2 A Thought Experiment for Retrocausality

Consider a Schrödinger's cat experiment in the presence of a time machine. A cat is placed inside a box with a radioactive isotope. A detector will release a poison if the isotope decays. There exists a clock stationary in the box' lab frame which measures lab time. The isotope is removed after a duration of time such that there is a 50% chance of the cat being poisoned. The isotope is removed automatically from the box at lab time t_0. Then the box is opened at $t_1 > t_0$ and the cat is observed to be alive or dead. After that, the observer uses the time machine to travel back in time. In the past, he opens the box at lab time t' such that $t_0 < t' < t_1$. The isotope was already removed from the box at t_0 so the poison was either released or not before t'. If wavefunction collapse does not have retrocausal effects, there should be a 50% chance of finding the cat either alive or dead at t' despite the cat being found in one state or the other at t_1. The theory of quantum mechanics predicts that the collapse of the cat's wavefunction to the alive or dead eigenstate at t_1 should not effect the probability for observing one state or the other in the past at t' but theory alone is not sufficient to determine the outcome of an experiment. It is possible that real time machine experiments would show that if the cat is observed to be alive or dead at t_1, then opening the box at t' will always yield a like result. The interpretation would be that the life or death was decided before t_0 when the isotope was removed.

In that case, quantum theory would have to concede the retrocausal effects disputed in [85]. Without doing the experiment, there is no way to know what would be the result. Even if the result of the experiment showed that the cat's state at t' does not universally agree with the state at t_1, the many worlds interpretation of QM would still make it impossible to conclude that the cat was a superposition prior to the respective measurements.

1.8.3 The Collapse Problem

The issue of collapse is mysterious independently from any questions about causality. How *exactly* does a detector put a superposition quantum state into an eigenstate? Neither quantum theory nor its interpretations offer a good answer to this question.

It is intrinsic to QM that observables are represented by Hermitian operators. Once that is established, mathematical collapse by projection follows directly. However, the axiom that a physical detector should be represented by a non-physical instantaneous collapse operator is unsatisfying. If an operator acts on a non-eigenstate at time t_0 and an eigenstate is instantaneously output, one could ask about the state at t_0 and get two good answers, or two bad ones. Although we might make an appeal to uncertainly in the experimental resolution of time, QM is a theory of states in Hilbert space *at a definite time*. Is the state at t_0 collapsed or diffuse? Is it semi-diffuse? State reduction is a discontinuous mathematical operation but an appeal to the $\Theta(t-t_0)=\frac{1}{2}$ property of the Heaviside function cannot tell us anything about the physics at t_0 because the theory of linear operators does not permit halfway collapse in progress. So, it is disappointing that QM provides no equations of motion such that diffuse, unmeasured superposition states might evolve smoothly into sharp, measured eigenstates. Due to Schrödinger's equation being a heat equation, Schrödinger evolution can only broaden probability distributions. It can never narrow them. This flies in the face of what is observed: wavefunctions diffuse and then they collapse. Something more than $\mathcal{H}\to\mathcal{H}$ Schrödinger evolution must take place between consecutive measurements. The intermediate steps of $\mathcal{H}\to\Omega\to\mathcal{A}\to\mathcal{H}$ are introduced to accommodate a theoretical structure for that additional process. Two extra steps will allow us to add one step of new physics and a second step to ensure that the new physics arrives at the known result, albeit with a better explanation than QM provides.

In general, the action of an observation on a quantum state is a projection into one of the corresponding operator's eigenstates. However, the action of that operator on a state is

$$\hat{A}|\psi\rangle = \hat{A}\sum c_n|a_n\rangle = \sum c_n a_n|a_n\rangle \quad . \tag{1.8.1}$$

This has not executed the projection operation. Namely, a measurement of observable A should be

$$\hat{P}_k\big|\psi\big\rangle = \big|a_k\big\rangle \quad , \tag{1.8.2}$$

so that if eigenvalue a_k is obtained from the first measurement, any number of rapidly repeated measurements will also yield a_k. In (1.8.2), $\hat{P}_k$ has *projected* ψ into the 1D eigenspace spanned by $|a_k\rangle$. Unfortunately, there is no dynamical equation for this and we must say "this is where the magic happens." Only *after* finding eigenvalue a_k, collapse is implemented by operating with

$$\hat{P}_k = \frac{1}{c_k}\big|a_k\big\rangle\big\langle a_k\big| \quad . \tag{1.8.3}$$

This extra step at the end of a time evolution is unnatural and clunky but it is the best QM has to offer for a mathematical description of wavefunction collapse.

Regarding two events a and b and their corresponding measurements A and B, it is known that the states observed at A and B cannot be δ functions. δ functions are not valid wavefunctions in the sense of the Born interpretation which says that a wavefunction's modulus squared is a real number. δ functions are also non-compliant with Heisenberg uncertainty. However, it is an open question of ontology and/or epistemology whether or not δ functions are part of the process. QM says nothing about whether a physical detector forces a quantum state into a mathematically singular δ function at a or b, or only into the width of an experimental resolution. To wit, there exist two position operators: $\hat{x}$ and $\hat{X}_{x_1}^{x_2}$. The first asks where the particle is and the second asks if the particle is between x_1 and x_2. Projection onto an eigenstate of $\hat{x}$ kicks the state out of Hilbert space as

$$\hat{P}_{\hat{x}} : \underbrace{\left\{\int dk\, A(k)\, e^{i(kx-\omega t)}\right\}}_{\text{wavepackets}} \quad \to \quad \underbrace{\big\{\delta(x-x_0)\big\}}_{\text{eigenstates of } \hat{x}} \quad . \tag{1.8.4}$$

This means that projection onto an eigenstate of $\hat{x}$ at a or b cannot possibly return the narrowly peaked wavepacket observed at A or B. In terms of the RHS $\{\mathcal{H}', \mathcal{A}', \Omega'\}$, (1.8.4) reads as $\hat{x} : \mathcal{H}' \to \Omega'$. On the other hand, $\hat{X}_{x_1}^{x_2}$ is such that

$$\hat{P}_{\hat{X}} : \underbrace{\left\{\int dk\, A(k)\, e^{i(kx-\omega t)}\right\}}_{\text{wavepackets}} \quad \to \quad \underbrace{\left\{\int dk\, A(k)\, e^{i(kx-\omega t)}\right\}}_{\text{eigenstates } \hat{X}_{x_1}^{x_2}} \quad . \tag{1.8.5}$$

Since a physical measurement can never give us more information than whether or not a particle is found in some region, $\hat{X}_{x_1}^{x_2}$ represents a physical measurement while $\hat{x}$ does not. So, there exists an important, open question about what is really going on at a and b.

In the psychological picture of the MCM, the observer learning that the particle is or is not in a given region is the measurement A or B, not the event a or b. δ-valued states are not observable and the question of the unobserved state at a or b remains open: does the wavefunction collapse to a δ function between A and B, or not? Although the eigenstates of $\hat{x}$ are not observable, do they correspond to the results of the state interacting with a detector at the events a and b? The inability of QM to answer this question is referenced when it is asked if QM might be a hack. We would like the theory to tell us about a and b but it only tells us about A and B. QM works around the deeper issue regarding fundamental interactions while answering the practical question about what is visible. A quantum mechanic from Copenhagen might argue that asking about the ontological realism of a state away from A or B is a blunder because that knowledge does not exist but the truth is only that such knowledge does not exist in QM. It might exist and another theory might describe it. The three-fold process of $\hat{M}^3$ is formulated to add resolution to this gray area. Under $\hat{M}^3$, an event happens at a, the results of which are observed at A. Then one predicts what will happen at b, waits for b to happen, and then observes the results of b at B.

We want to know how probability distributions can become narrower when the Schrödinger equation only broadens them, and we want to know if they become singularly narrow as δ functions at some point during the transit of the unit cell. If there is a layer of quantum theory where δ functions are obtained, that layer would be uniquely well suited to connections with the theory of test masses moving along geodesics in relativistic spacetime because GR is a theory of points, or position eigenstates. Thus, the MCM's three-fold structure is well suited to answering such questions about the separateness of the a, b event layer and the A, B observation layer. Suggesting the relevance of the lag between the two, the MCM prediction that observables should be correlated with the delay between an event and its measurement was confirmed in BaBar's observation of time reversal symmetry violation (Section 0.1) [32]. Such delay effects are consistent with a state collapsing to a δ function at an event and then returning to the Hilbert space as a wavepacket when the observer is eventually notified of the event.

Due to Weyl's criterion, the eigenstates of an operator with a continuous spectrum can be approximated to arbitrary precision by the states in the operator's domain of self-adjointness.[1] Referring to (1.8.4), Weyl's criterion says that a δ function may be well approximated by wavepackets from the Hilbert space. If one substitutes the approximate eigenvectors for the real eigenvectors, the $\hat{P}_{\hat{x}}$ projection operator can output the state which is observed at A or B. In this approximation, $\hat{P}_{\hat{x}}:\mathcal{H}'\to\mathcal{H}'$ does not kick states out Hilbert space and there is no inherent appeal to rigged Hilbert space. However, the method of approximate eigenvectors identifies a,b with A,B when the real time lag between them leaves room for additional physics. QM has little or nothing to say about this lag and the Weyl convention for approximate eigenvectors presumes its non-existence.

It is acutely important for the MCM whether or not the state actually collapses to a δ function so we must not preclude the possibility for δ functions to appear in the chirological time evolution of a state from $\mathcal{H}_k$ to $\mathcal{H}_{k+1}$. For instance, if there are no δ-valued states during a transit of the unit cell, then there is no place in the theory for states unique to the Ω' part of rigged Hilbert space. In turn, this will affect the MCM scheme of fundamental particles because the three generations of matter particles are (presently) associated with the three RHS state spaces. A distinct and potentially useful property of the Ω' states is that a wavepacket $u(x,t)\in\mathcal{H}'$ will thermalize, or diffuse, such that

$$\big|\psi,t_0\big\rangle = c_0(t_0)u_0(x) \quad\longrightarrow\quad \big|\psi,t_1\big\rangle = \sum_j c_j(t_1)u_j(x) \quad , \tag{1.8.6}$$

but if that same state is moved into Ω'

$$\psi\in\mathcal{H}'\subset\Omega' \quad\Longrightarrow\quad \psi\in\Omega' \quad , \tag{1.8.7}$$

we should expect distinct thermalization behavior:

$$\big|\psi,t_0\big\rangle = c_0(t)u_0(x) \quad\longrightarrow\quad \big|\psi,t_1\big\rangle = \sum_j c_j(t_1)u_j(x)+\sum_k c_k(t_1)\delta(x-x_k) \quad .^2 \tag{1.8.8}$$

Such behaviors might be observably correlated with correlation amplitudes.

δ functions are also desirable for applications toward quantum gravity. The layer of collapse to a δ function at a,b is well suited to communication with GR because GR is a theory of points in spacetime. Points are exact time and space eigenstates, not approximate ones. Although it is required to describe measurements with $\hat{X}_{x_1}^{x_2}$, an

[1] Unbounded operators such as $\hat{x}$ and $\hat{p}$ are typically not self-adjoint on all of Hilbert space. The subspace of Hilbert space on which an operator is self-adjoint (Hermitian) is the main limitation selecting the $\mathcal{H}'$ subspace of Hilbert space $\mathcal{A}'$ as the space of physical states in RHS. Since physical observables are represented in QM by self-adjoint operators, physical states must reside within an operator's domain of self-adjointness.

[2] For convenience, we use discrete notation for continuous states.

association of events with $\hat{x}$ introduces a layer where the objects of quantum theory are mathematically compatible with the objects in the theory of gravitation. For this reason and others, the MCM mechanism for wavefunction collapse should aim to produce mathematically singular δ functions.

1.8.4 The Double Slit Experiment

The double slit experiment is depicted in Figure 11. In Section 1.8.5, we will examine the problem of the wavefunction collapsing to a point of scintillation at t_s. In this section, we will examine only the destruction of the interference pattern on the screen when the path through the slits is measured.

With slits labeled R and L, the MCM proposal to explain the observed wave-particle duality [70] is

$$\text{Waves} \longrightarrow \begin{cases} \big|\psi_R, t_0; \hat{\pi}^0\big\rangle \to \big|\psi_R, t_p; \hat{\pi}^0\big\rangle \to \big|\psi_R, t_s; \hat{\pi}^1\big\rangle \\ \big|\psi_L, t_0; \hat{\pi}^0\big\rangle \to \big|\psi_L, t_p; \hat{\pi}^0\big\rangle \to \big|\psi_L, t_s; \hat{\pi}^1\big\rangle \end{cases} \tag{1.8.9}$$

$$\text{Particles} \longrightarrow \begin{cases} \big|\psi_R, t_0; \hat{\pi}^0\big\rangle \to \big|\psi_R, t_p; \hat{\pi}^1\big\rangle \to \big|\psi_R, t_s; \hat{\pi}^2\big\rangle \\ \big|\psi_L, t_0; \hat{\pi}^0\big\rangle \to \big|\psi_L, t_p; \hat{\pi}^0\big\rangle \to \big|\psi_L, t_s; \hat{\pi}^1\big\rangle \end{cases} \quad . \tag{1.8.10}$$

A first measurement regards the preparation of a monochromatic particle beam at t_0. This measurement takes place in $\mathcal{H}_0$ so the state of a particle at the source is

$$\big|\psi, t_0; \hat{\pi}^0\big\rangle = \frac{1}{\sqrt{2}}\big|\psi_R, t_0; \hat{\pi}^0\big\rangle + \frac{1}{\sqrt{2}}\big|\psi_L, t_0; \hat{\pi}^0\big\rangle \quad . \tag{1.8.11}$$

The total probability amplitude is the sum of the amplitudes for going through the upper and lower slits. At t_p, the beam hits the diffraction plate. Then it continues as ψ_R and ψ_L waves having slits R and L as their respective sources. If no measurement is made at the slits, each will emit a wavefront of probability amplitude on the $\hat{\pi}^0$ level of aleph. (Recall that $\hat{\pi}$ levels of aleph enumerate successive measurements.) Since ψ_R and ψ_L are monochromatic and on the same level of aleph, they are not orthogonal. The waves will interfere and a subsequent measurement at t_s will never show a particle arriving on the screen at the minima between interference fringes. Many repetitions will show that the probability distribution on the screen is consistent with interference between wavefronts sourced from R and L, as in (1.8.9).

Early attempts to explain wave-particle duality in the double slit experiment resulted in the uncontrollable disturbance hypothesis. It was supposed that the act of

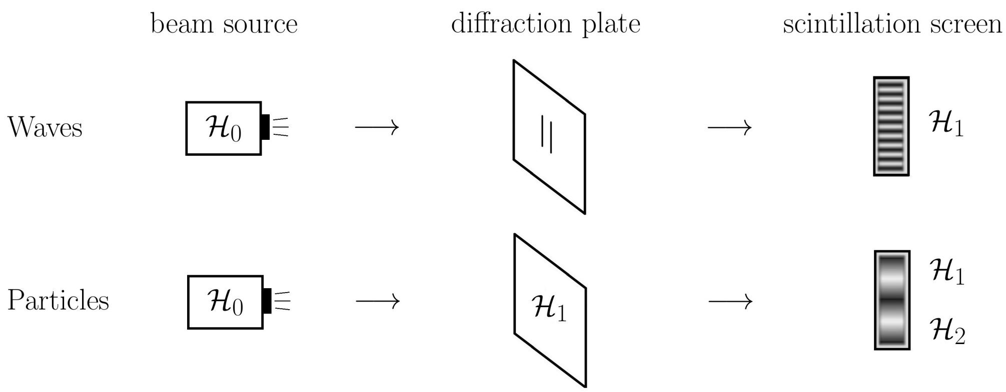

Figure 11: Above, measurements at the source and screen are taken in $\mathcal{H}_0$ and $\mathcal{H}_1$. No measurement is made to determine which slit the particle passed through. After many repetitions, wave interference is observed on the screen because the monochromatic wavefronts emanating from each slit are on the same level of aleph. Below, three measurements are made. In addition to looking at the source and screen, the observer determines which slit the particle passes through. After many repetitions, wave interference is not observed because the intermediate measurement increased the level of aleph for the wavefront coming through one slit or the other. Waves on different levels of aleph cannot form interference patterns because they are orthogonal.

measurement cannot be *ideal* and that, therefore, the measurement interaction between two quantum systems adds an unobservable phase to the observed state: ψ_R or ψ_L. In turn, that phase destroys the interference pattern. The uncontrollable disturbance explanation has not panned out and increasingly complicated workarounds were formulated so as to avoid the conclusion that the particle knows about what will happen at t_p. However, the double slit experiment is very strange and no one understands it. It is hard to avoid the conclusion that the particle somehow *knows* whether or not an observer will determine the path through the diffraction plate. If a position measurement is made at t_p, it is *usually* said that the particle knows to go through one slit or the other. As a result, the interference pattern is destroyed due to the lack of any wave emittance from the other slit. This explanation is unsatisfying because the particle should not know anything other than to obey the action principle.

A superior MCM explanation for the observed phenomenon is that the particle *always* goes through both slits [70]. Rather than knowing what the observer will do at t_p, a measurement at the diffraction plate separates ψ_R and ψ_L onto different levels of aleph. The interference pattern is destroyed because orthogonal plane waves cannot interfere, as in (1.8.10). (Orthogonal plane waves were developed in Section

1.7.3.) The problem which remains is to formulate a mechanism by which the collapse associated with an intermediate measurement at t_p will separate ψ_R and ψ_L onto two different levels of aleph. An alternative mechanism might invoke the action associated with a transit of the unit cell so that the particle choosing one slit or another in the presence of an intermediate measurement *does* reflect the action principle. Crossing an extra unit cell would have higher action favored by the maximum action principle. In the remainder of this section, however, we will consider the former process from [70] in which the particle always goes through both slits.

The MCM plane wave ansatz is

$$\psi(\mathbf{x},t,\chi^4)=e^{i(\mathbf{k}\cdot\mathbf{x}-\omega t+\beta\chi^4)} \quad . \tag{1.8.12}$$

The double slit application requires β whether or not χ^4 is dimensionless. To use the rules for orthogonal plane waves (Section 1.7.3), we must encode the level of aleph onto the χ^4 part of the argument with β. Following the usual notation for

$$\phi(\mathbf{x},t)=A_0e^{ik_\mu x^\mu}=A_0e^{i(\mathbf{k}\cdot\mathbf{x}-\omega t)}\equiv\big|k_\mu\big\rangle \quad , \tag{1.8.13}$$

we will write the ansatz as

$$\psi(\mathbf{x},t)=A_0e^{ik_Ax^A}\equiv\big|k_A\big\rangle \quad . \tag{1.8.14}$$

The minus sign on ωt in (1.8.13) follows from the $\{-+++\}$ metric signature in $\mathcal{H}$. The sign on χ^4 in (1.8.14) will depend on the $\{-+++\pm\}$ metric signature in $\Sigma^\pm$. Intermingling for simplicity the abstract and physical coordinates, and ignoring the constant A_0, the orthogonality of MCM plane waves follows as

$$\big\langle k'_A\big|k_A\big\rangle=\underbrace{\int_{-\infty}^{\infty}dt\,e^{-i(\omega-\omega')t}}_{\delta(\omega'-\omega)}\underbrace{\iiint d^3x\,e^{i(\mathbf{k}-\mathbf{k}')\cdot\mathbf{k}}}_{\delta^{(3)}(\mathbf{k}-\mathbf{k}')}\int_{-\infty}^{\infty}d\chi^4\,e^{i(\beta-\beta')\chi^4} \quad . \tag{1.8.15}$$

If the χ^4 part is like a small box plane wave (Section 1.7.3), β should be discrete and the integral over χ^4 becomes the Kronecker δ. If β is continuous, it becomes the Dirac δ. The case of discrete β_n lends itself directly to the identification of lattice sites or levels of aleph. Since each piecewise $\chi^4_\pm$ or $\chi^4_\varnothing$ has its origin in a given brane with a corresponding scale, β_n would be a scale factor used to preserve the notion of disparate relative scale between levels of aleph. This scale must be considered when taking the inner product of states on different levels of aleph. For instance, the inner

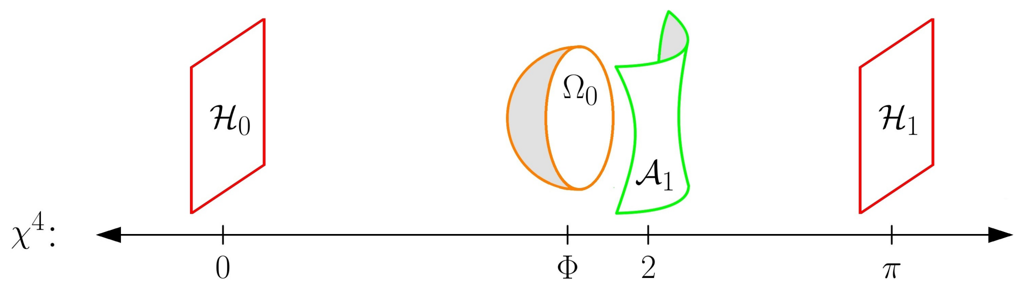

Figure 12: The monotonic increase of $|\hat{e}_\mu|$ across the unit cell's labeled branes suggests a continuum of scale factor β.

product of states in the μ- and ν-branes on the m and n levels of aleph would be

$$\big\langle k'_A \big| k_A \big\rangle = \big\langle \psi'; \hat{e}^n_\nu \big| \psi; \hat{e}^m_\mu \big\rangle = \underbrace{\int d^4x \, e^{i(k_\lambda - k'_\lambda)x^\lambda}}_{\langle \psi' | \psi \rangle} \underbrace{\int_{-\infty}^{\infty} d\chi^4 \, e^{i(|\hat{e}_\mu|^m - |\hat{e}_\nu|^n)\chi^4}}_{\delta_{\mu\nu}\delta_{mn}} \quad . \tag{1.8.16}$$

We have previously obtained the relative scale $2\pi\Phi$ between two $\mathcal{H}$-branes as the increase of scale at each labeled brane by an amount proportional to the magnitude of its ontological specifier, as in Figure 12. The β in (1.8.16) imposes that relative scale on χ^4 in the plane wave state: the scale is the absolute value of $\hat{e}^m_\mu$. Considering the big box case of unbounded plane waves, however, β is continuous rather than discrete. The continuum of scale factor is also seen in Figure 12. Using notation in which the continuous χ^4 parameter associated with the primed level of aleph is $\chi' = \beta'\chi^4 + c'$, we would write

$$\big\langle \psi'; \hat{e}_\nu \big| \psi; \hat{e}_\mu \big\rangle = \big\langle \psi' \big| \psi \big\rangle \underbrace{\int_{-\infty}^{\infty} d\chi^4 \, e^{i(\chi' - \chi'')}}_{\delta(\beta' - \beta'')} \quad . \tag{1.8.17}$$

Either of (1.8.16) or (1.8.17) is sufficient to motivate the wave-particle duality observed in the double slit experiment. All that is required for (1.8.9) and (1.8.10) is that β identifies the level of aleph and that waves on different levels of aleph are orthogonal. With this condition written plainly, future work must devise a mechanism by which a measurement at t_p will separate ψ_R and ψ_L onto different levels of aleph.

1.8.5 An Application for the Theory of Negative Time

As state reduction (wavefunction collapse) is understood in the present theory, a measurement at time t_0 is essentially such that

$$\dot{\psi}(t_0) = \infty \quad . \tag{1.8.18}$$

This is inherently problematic because ∞ is analytically intractable and $\dot{\psi}$ obeys

$$i\hbar\dot{\psi} = \hat{H}\psi \quad . \tag{1.8.19}$$

If $\dot{\psi} = \infty$, Schrödinger's equation is only satisfied with unphysical, infinite energy. Furthermore, the time arrow is such that infinite $\dot{\psi}$ will cause total decoherence of the wavefunction rather than total collapse. In this section, we will sketch a theoretical mechanism for the apparent infinite rate of wavefunction collapse, and for a period of Schrödinger coalescence following the usual period of Schrödinger diffusion.

Even without a diffraction grating between a source and a scintillation screen, it is not known how the wavefunction might undergo smooth diffusion in transit and then suddenly collapse to a point on the screen. Wavepackets evolving under the Schrödinger equation can only become broader, never narrower. It is also not known whether the wavefunction collapses to a δ function on the screen or only down into the region spanned by a finite spot of scintillation. The state's confinement to the spot may be better associated with the time that the scintillation photons reach the observer than it is with the interaction between the beam and the screen. The state at the time of that interaction, b as opposed to B, may be a δ function. So, despite the observer's knowledge being limited by the experimental resolution, we might ask what the wavefunction is *really* doing on the screen. Is it proper to consider a theory in which the observed interaction between the screen and the particle outputs a δ function? While the answers to such questions are not known, it is known that nothing more than one's preference supports the argument against asking what is *really* happening in QM.[1]

QM is such that the wavefunction obeys Schrödinger's equation at all times except when measurements are made. There, singular, instantaneous collapse flies in the face of all other known physical processes. Fractional distance analysis offers new tools for recasting $\dot{\psi}=\infty$ as another expression not at odds with physics as usual. Any rate of collapse in the neighborhood of infinity must be observationally indistinguishable from an infinite rate of collapse so we may replace $\dot{\psi}=\infty$ with

$$\dot{\psi} \in \widehat{\mathbb{R}} \quad \implies \quad \dot{\psi}(t_0) = \aleph_{\mathcal{X}} + b \quad . \tag{1.8.20}$$

($\widehat{\mathbb{R}}$ is the positive branch of $\mathbb{R}$ less the neighborhood of the origin and the non-arithmatics, as in Section 1.6.1.) With this rate of collapse, $i\hbar\dot{\psi}(t_0)=\hat{H}\psi(t_0)$ implies

[1] 't Hooft's non-MCM cellular automata model of QM addresses similar questions about what really happens in QM with novel objects such as *ontological states*, *beables*, and *changeables*. The cellular automata model is parsed for future inquiry in Section 58.

a finite Hamiltonian:

$$\left.\begin{array}{l} |\psi|^2 \leq \infty \\ \dot{\psi}(t_0) \in \widehat{\mathbb{R}} \end{array}\right\} \quad \Longrightarrow \quad \left|\hat{H}\psi\right| \in \widehat{\mathbb{R}} \quad . \tag{1.8.21}$$

Energy in the neighborhood of infinity is consistent with the principle of maximum action discussed in Section 1.5. Presuming free space between a beam source and a scintillation screen, energy in the neighborhood of infinity requires that we write

$$\hat{H} = \hat{H}_0 + \hat{H}_{\text{int}} \quad , \tag{1.8.22}$$

in which the interaction energy associated with $\dot{\psi}\in\widehat{\mathbb{R}}$ vanishes everywhere except for the time and place of a measurement. $\hat{H}_{\text{int}}$ should not be a function of the chronological time because the observer may choose to make a measurement at arbitrary times. Such a function could not be defined until after t_0 was chosen. Referring back to the double slit experiment, $\hat{H}_{\text{int}}$ cannot be a function of the spatial variables alone because collapse only happens at the spatial position of the slits when a measurement is made. Therefore, $\hat{H}_{\text{int}}$ should be a function of the chirological time such that the arbitrary $t=t_0$ is associated with a regularized periodicity in χ^4. For example, we have associated the preparation of a beam with measurement A in $\mathcal{H}_0$ so the observation of a subsequent scintillation spot in the path of the beam should be associated with measurement B in $\mathcal{H}_1$. Between A and B, event b must occur: the interaction of the beam and the scintillator. We will take that as the place where $\dot{\psi}$ suddenly becomes very large. This sudden change of scale in $\dot{\psi}$ is well associated with the change of the level of aleph at $\varnothing$ which is located at a constant abstract distance between successive $\mathcal{H}$-branes. Perhaps we might associate event b with the $\varnothing$-brane and set a δ-like $\hat{H}_{\text{int}}$ term at the location of that topological obstruction between Ω and $\mathcal{A}$. A δ function is a good candidate for an energy that vanishes everywhere except for the event of wavefunction collapse when it becomes infinite or enters the neighborhood of infinity.

Now we have suggested a method by which one might obtain the large $|\dot{\psi}|$ observed in experiments but it remains to explain the sign on $\dot{\psi}$. For that, we will refer to the theory of negative time. A nice application will be to implement dynamical collapse as a step of reversed time evolution in $\hat{M}^3$ through a region with a reversed time arrow such as Σ^-. (We may also introduce a reversed time arrow in the $\chi^4_\varnothing$ coordinates between $\varnothing$ and $\mathcal{A}$ if needed.) Diffusion by Schrödinger evolution in positive time will become coalescence in negative time, as is required for wavefunction collapse. Since this step occurs on the higher level of aleph associated with $\Sigma^-_{\{k+1\}}$, we may appeal to the scale of that level of aleph generating the appearance of discontinuous, non-dynamical collapse as observed from the lower level. The problem of $\dot{\psi}\notin\mathbb{R}_0$ might be further simplified through an appeal to infinite relative scale between two levels of aleph. Using $\dot{\psi}_{\{k\}}$ to refer to the rate of change given in the scale of $\mathcal{H}_k$, we may obtain

$$\dot{\psi}_{\{0\}}=\aleph_{\mathcal{X}}+b\quad\longrightarrow\quad\dot{\psi}_{\{1\}}=\frac{\dot{\psi}_{\{0\}}}{\aleph_{\mathcal{Y}}}=\frac{\mathcal{X}}{\mathcal{Y}}\quad.^{1}\tag{1.8.23}$$

Here, we assume that normalization of the observer's reference frame onto the $k=1$ level of aleph requires division by $\aleph_{\mathcal{Y}}=\mathcal{Y}\widehat{\infty}$. This choice of scale, or a similar one, makes it possible to resolve the apparent instantaneous rate of change observed on one level of aleph as a rate in the neighborhood of the origin on the other level of aleph.

For the present theoretical application, we must refer to the original picture of $\hat{M}^3$ executing $t_0\to t_{\text{max}}\to t_{\text{min}}\to t_1$ where $t_1=(t_0+\Delta t)$ [30]. We will identify this process with the current one so that Ω is associated with t_{max} and $\mathcal{A}$ is associated with t_{min}. Essentially, we will require that $\varnothing$ is the same big bounce separating two cycles of cosmology and that it can be reached in the x^0 direction or the χ^4 direction. This as fits an interpretation of $\varnothing$ as a black brane or a black hole/white hole pair.[2] Having established this identification of paths in principle, we will examine the evolution of a wavepacket across the unit cell parameterized with the chronological time. The steps of $\hat{M}^3$ will be taken as $t_0\to\widehat{\infty}$, $\widehat{\infty}\to-\widehat{\infty}$, and $-\widehat{\infty}\to(t_0+\Delta t)$.

Given a δ function initial condition at a $t=0$ in $\mathcal{H}_0$, a particle subjected only to the free particle Hamiltonian $\hat{H}_0$ evolves as

$$\psi(x,t)=\begin{cases}\delta(x) & \text{for}\quad t=0\\ \sqrt{\dfrac{m}{2\pi\hbar t}}\exp\left(\dfrac{-i\pi}{4}\right)\exp\left(\dfrac{imx^2}{2\hbar t}\right) & \text{for}\quad t>0\end{cases}\quad.\tag{1.8.24}$$

When the wavepacket gets to Ω or $\varnothing$ associated with chronological timelike infinity, we have

$$\psi(x,\widehat{\infty})=\sqrt{\frac{m}{2\pi\hbar\widehat{\infty}}}\exp\left(\frac{-i\pi}{4}\right)\exp\left(\frac{imx^2}{2\hbar\widehat{\infty}}\right)=0\quad.^{3}\tag{1.8.25}$$

[1] Arithmetic axioms for numbers in the neighborhood of infinity [2] are such that $(\aleph_{\mathcal{X}}+b)/(\aleph_{\mathcal{Y}}+c)=\mathcal{X}/\mathcal{Y}\in\mathbb{R}_0$. The loss of information about b in (1.8.23) may have applications toward information loss in quantum processes which exceed the scope of the present section.

[2] If the periodicity of x^0 associated with cosmological bouncing sets the x^0 axis as a great circle of a sphere, the periodicity on χ^4 is necessarily more complicated than a second great circle. Great circles of a sphere intersect twice but we desire that chronos and chiros should intersect at the past and future bounces, and in the present. Scribing this triple intersection onto a sphere gives chiros a character of *chirality* or helicity relative to chronos.

[3] $\psi=0$ does not satisfy the $\langle\psi|\psi\rangle=1$ probability condition. We might avoid this problem by citing the zero volume

This final state demonstrates an important difference between the wave equation and the heat equation: the wave equation can recover initial conditions by reversing time but it is impossible to recover the initial conditions by reversing the heat equation. Starting with $\psi(x,-\widehat{\infty})=0$ as the initial condition for a final leg of $\hat{M}^3$ will not result in a recondensed δ function if $\dot{\psi}(x,-\widehat{\infty})=0$, which is the present case. To reconstitute a δ function by reverse time Schrödinger evolution from the $\psi=0$ initial condition, a non-vanishing $\dot{\psi}$ initial condition is required. As a matter of simulating this condition with numerical analysis, we should consider the backward difference formula approximation for the first derivative:

$$\dot{\psi}(x,t)=\frac{\psi(x,t)-\psi(x,t-\delta t)}{\delta t} \quad . \tag{1.8.26}$$

By imposing the condition that $\psi(x,t-\delta t)$ was non-zero, meaning that the wavefunction did not become identically zero until the last step of a Gaussian integration to infinity, we will obtain a non-zero $\dot{\psi}$ initial condition for the $\hat{M}^3$ step of $-\widehat{\infty}\to t_0+\Delta t$. We will use the $\hat{M}^2$ step of $\widehat{\infty}\to-\widehat{\infty}$ to reverse the sign on $\dot{\psi}$. A cursory examination of (1.8.24) shows that $\psi=0$ for any $t\notin\mathbb{R}_0$ so the $\hat{M}^1$ step should evolve ψ only to the end of the neighborhood of the origin. Evidently, we are working in the coordinates where $\widehat{\infty}$ is identified with $\mathcal{F}_0$, as in Section 1.6.8 (Figure 9). If we identify $\widehat{\infty}$ with the first time beyond time in the neighborhood of the origin, then $\psi(x,t-\delta t)\neq 0$ and the backward difference formula for the derivative will facilitate reconstitution of the δ function beyond infinity. That step will begin with a non-vanishing first derivative.

The method described above will move $\delta(x)$ in $\mathcal{H}_0$ to $\delta(x)$ in $\mathcal{H}_1$. However, the chronological time in $\mathcal{H}_1$ is ${}_0t+\Delta t$. Decoherence occurs in the time interval $(t_0,t+\Delta t)$ so if a detector collapsed the state to $\delta(x)$ at event a, the detector should collapse it to $\delta(x\pm\Delta x)$ at event b. Uncertainty is such that repeated measurements of position should differ somewhat. As a proposal for obtaining the Δx spatial variation needed for agreement with experiments, we will refer to the irrational part of the relative scale between levels of aleph. This is the $2\pi\Phi$ appearing in $\hat{M}^3|\psi;\hat{\pi}^0\rangle=2\pi\Phi|\psi;\hat{\pi}^1\rangle$ (wherein infinite relative scale may be implicit in $\hat{\pi}^k\to\hat{\pi}^{k+1}$.) When rescaling the observer's frame onto a new level of aleph as in (1.8.23), and when the scale is an irrational number, we may achieve wavefunction decoherence leading to $\delta(x\pm\Delta x)$ as a novel numerical effect.

Due to an inability to exactly represent irrational numbers as floats, it will not be possible to exactly reverse diffusion in Σ^+ with coalescence in Σ^- when x is altered

of $\varnothing$ in the physical coordinates. Even if ψ did not equal zero, the integral over a pointlike $\varnothing$ singularity would not be equal to unity. However, we might appeal to the changing level of aleph to resolve the point as a volume.

by an irrational scale factor. For instance, we have shown that the coordinate transformations between the $\{x_+^\mu, x^\mu, x_-^\mu\}$ physical coordinates are all such that the entries in the transformation matrix are real numbers but the relationship that sets $t=\widehat{\infty}$ as a conformal infinity at $\chi_+^4=\Phi$ is likely to have a function in it, e.g.:

$$t(\chi_+^4) = \tan\left(\frac{\pi\chi_+^4}{2\Phi}\right) \quad\Longrightarrow\quad t(\Phi)=\infty \quad . \tag{1.8.27}$$

In turn, the transformation matrix between physical and abstract coordinates will have non-constant function entries whose chain rule properties under differentiation are much different than the static scale factors among the different branes' physical coordinates. Upon irrational rescaling, the appearance of Φ in the argument of functions periodic in 2π will inevitably require float-precision approximations. The associated rounding error might be useful for producing what QM assigns as stochastics to dynamics in the MCM. $\delta(x)$ in $\mathcal{H}_0$ will be reconstituted as $\delta(x\pm\Delta x)$ on $\mathcal{H}_1$ simply due to rounding error even if the Gaussian time steps are exactly reversed. Furthermore, when Φ appears in the periodic argument of functions such as e^x, the accumulation of rounding error across many levels of aleph will never lead to runaway, unphysical solutions because the error will be taken modulo the period. The rounding error pushed through the function's periodicity may lead to behaviors similar to single slit diffraction particles appearing randomly on a scintillation screen. One would attempt to the write the correlation function describing rate of decoherence of a wavefunction between t and $t+\Delta t$ in terms of the rounding error. The language of Lyapunov exponents may be appropriate for such a characterization because chaos is a byproduct of determinism.

1.9 The Fine Structure Constant

Dirac is quoted as saying the origin of the fine structure constant is, "the most important unsolved problem in physics," and rightly so. The link between electromagnetism, special relativity, and quantum theory given by the inclusion of e, c, and $\hbar$ in

$$\alpha_{\text{QED}} = \frac{e^2}{4\pi\varepsilon_0\hbar c} \quad , \tag{1.9.1}$$

is a tantalizing hint of some fundamental unification which has escaped detection in prevailing theories. In that vein, Feynman wrote the following [86].

> "It is a simple number that has been experimentally determined to be close to 0.08542455. (My physicist friends won't recognize this number, be-

cause they like to remember it as the inverse of its square: about 137.03597 with about an uncertainty of about 2 in the last decimal place. It has been a mystery ever since it was discovered more than fifty years ago, and all good theoretical physicists put this number up on their wall and worry about it.) Immediately you would like to know where this number for a coupling comes from: ***is it related to pi*** [*emphasis added*] or perhaps to the base of natural logarithms? Nobody knows. It's one of the greatest damn mysteries of physics: a magic number that comes to us with no understanding by man. You might say the 'hand of God' wrote that number, and 'we don't know how He pushed his pencil.' We know what kind of a dance to do experimentally to measure this number very accurately, but we don't know what kind of dance to do on the computer to make this number come out, without putting it in secretly!"

The MCM value for the fine structure constant (FSC) is very much "related to π":

$$\alpha_{\mathrm{MCM}}^{-1} = 2\pi + \big(\Phi\pi\big)^3 \approx 137 \quad . \tag{1.9.2}$$

The original motivation for $\hat{M}^3$ in [30] was nothing more than a requirement to generate the $(\Phi\pi)^3$ term in $\alpha_{\mathrm{MCM}}^{-1}$. The other context for $\hat{M}^3$ and $\hat{\Upsilon} = \hat{\mathcal{U}} + \hat{M}^3$ was reverse engineered from that. (Appendix A reviews the original ideation for $\hat{M}^3$.) Since the subsequent introduction of the chirological variables has called into question the $\partial_x + \partial_t^3$ structure of

$$\hat{\Upsilon}\big|\Psi_\alpha\big\rangle = \big(\partial_x + \partial_t^3\big)\big|\Psi_\alpha\big\rangle = \alpha_{\mathrm{MCM}}^{-1}\big|\Psi_\alpha\big\rangle \quad , \tag{1.9.3}$$

in this section we will use $\hat{\alpha}$ such that

$$\hat{\alpha}\big|\Psi_\alpha\big\rangle = \alpha_{\mathrm{MCM}}^{-1}\big|\Psi_\alpha\big\rangle \quad . \tag{1.9.4}$$

Then we will return to $\hat{\Upsilon}$ in Section 1.11 and discuss its simultaneous roles regarding α_{MCM} and total evolution combining the chronological and chirological evolution operators $\hat{\mathcal{U}}$ and $\hat{M}^3$. A 0.4% discrepancy between α_{MCM} and α_{QED} is discussed in Section 1.9.4.

1.9.1 Fine Structure in the Unit Cell

The best way to find a place for $\hat{\alpha}$ and/or its eigenstate might begin with a survey of physics' existing roles for α_{QED}: the electron $g-2$, the Josephson junction, Sommerfeld's work regarding the *fine structure* splitting of atomic energy levels, etc. For

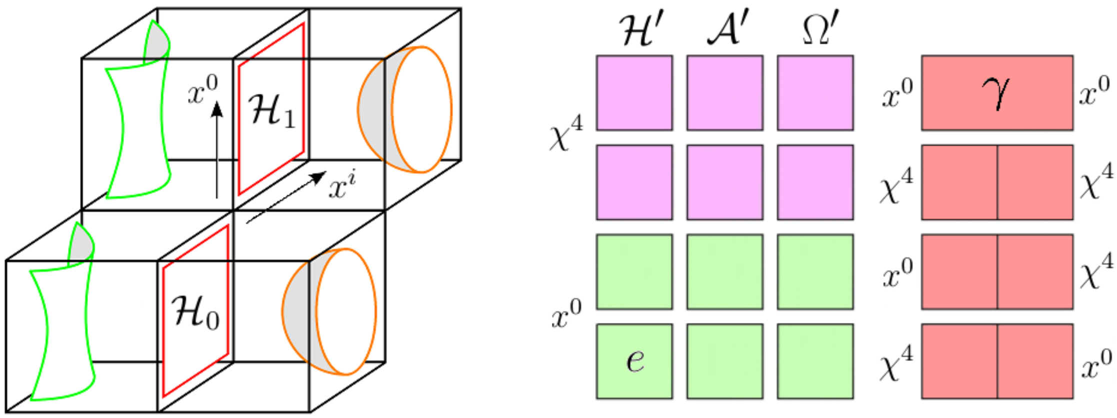

Figure 13: On the right, MCM fundamental matter particles are quanta of spacetime spanned by x^i (space) and either of x^0 or χ^4 (time). MCM fundamental bosons are constructed as connections of matter particles. On the left, the objects of the unit cell are easily parsed as two electrons and a photon. Each $\mathcal{H}$-brane is an x^0x^i quantum associated with the electron and the photon is formed as the union of two x^0x^i quanta. This arrangement of the unit cell emphasizes chronological continuity of x^0 between $\mathcal{H}_0$ and $\mathcal{H}_1$.

each given context, one would seek to amend existing relationships and interpretations with principles unique to the MCM.

The most elementary physical statement of the FSC is the ratio of two energies: the energy E_{ee} needed to close the distance d between two electrons and the energy E_γ of a photon with wavelength $\lambda = 2\pi d$:

$$\frac{E_{ee}}{E_\gamma} = \frac{\left(\dfrac{e^2}{4\pi\varepsilon_0 d}\right)}{\left(\dfrac{hc}{2\pi d}\right)} = \frac{e^2}{4\pi\varepsilon_0\hbar c} = \alpha \quad . \tag{1.9.5}$$

The MCM particle scheme in Figure 13 (also Section 0.3) is such that electrons are quanta of spacetime spanned by x^i and x^0. Photons are constructed from pairs of such quanta. Therefore, the E_{ee}/E_γ definition of α suggests the ratio of the energy between two $\mathcal{H}$-branes to the energy of a complete unit cell. As in Section 1.4, the $\{\mathcal{A}, \mathcal{H}, \Omega\}$ structure is evocative of the three spin states afforded to spin-1 vector photons. Even the $\{\Sigma^+, \varnothing, \Sigma^-\}$ structure suggests the massless photon's restriction to two polarization directions. Work is required to develop the MCM particle scheme to the point where more concrete statements can be made regarding the E_{ee}/E_γ ratio.

Furthermore, the hydrogen atom's electron and three nuclear quarks may be matched with an x^0x^i quantum and three $\{+,\varnothing,-\}$ variants of the χ^4x^i spacetime quantum. Since the hydrogen atom is foremost among α's physical settings, one would study the cases for the association of hydrogen's constructive elements with the structure of the unit cell. Particularly, we have associated the subscripting on χ^4 with QCD color charge so the up-up-down quark construction of the proton does not precisely match the three variants of χ^4. Instead, the particle scheme is such that the proton is constructed as two right-handed χ^4x^i spacetimes, and one left-handed. The intuitive association in Figure 13 is that the $\Sigma^\pm$ between two $\mathcal{H}$-branes combine with another instance of Σ^+ or Σ^- such that one has opposite helicity to the other two. Such issues remain to be studied and developed. A model of the unit cell as a 5D sphere whose radial direction is χ^4 is also in order.

1.9.2 The Fine Structure Constant as an Eigenvalue

The FSC is observable so it should be the real eigenvalue of a Hermitian operator $\hat{\alpha}$. An ansatz for $\hat{\alpha}$ is

$$\hat{\alpha}=\big(i\partial_0\big)+\big(i\partial_4\big)^3 \quad , \tag{1.9.6}$$

where

$$i\partial_0\big|\Psi_\alpha\big\rangle=2\pi\big|\Psi_\alpha\big\rangle \quad , \qquad \text{and} \qquad \big(i\partial_4\big)^3\big|\Psi_\alpha\big\rangle=\big(\Phi\pi\big)^3\big|\Psi_\alpha\big\rangle \quad . \tag{1.9.7}$$

The $i\partial$ operator is a sign conjugated momentum operator in the position representation or a position operator in the momentum representation. Such operators are Hermitian and the sum of two Hermitian operators is Hermitian.[1] It follows that $\hat{\alpha}$ is Hermitian. Its eigenvalue $2\pi+(\Phi\pi)^3$ is real so $\hat{\alpha}$ meets QM's minimum requirements for the operator representation of an observable.

Given the proposed form of $\hat{\alpha}$, the eigenstate with eigenvalue α^{-1}_{MCM} is

$$\Psi_\alpha(x^0,\chi^4)=\exp\big\{-i\big(2\pi x^0+\Phi\pi\chi^4\big)\big\} \quad . \tag{1.9.8}$$

The particle-in-a-box wavefunction used for Ψ_α in [3, 30] was not an eigenstate of $\hat{\alpha}$ but a former trivial deficiency is remedied in (1.9.8). Still, it remains to find the meaning of this Ψ_α state. Since it is our desire to associate the FSC with the structure of the unit cell, we should consider the case in which Ψ_α is a plane wave whose wave vector $\mathbf{k}$ or k_μ is an MCM reciprocal lattice vector in the MCM direct lattice. The

[1] Since the momentum operator is defined on an infinite-dimensional Hilbert space, the Hermiticity condition $\hat{\mathcal{O}}=\hat{\mathcal{O}}^\dagger$ is technically replaced with a broader condition of self-adjointness. This condition requires that operation to the right and operation to the left with the conjugate transpose produce the same result.

case in which Ψ_α is the state of the lattice rather than a state subjected to the lattice's regularity structure must be considered. Association of Ψ_α with the lattice itself will motivate a context for the $\hat{\Upsilon}$ total evolution operator to return a universal eigenvalue when it acts on Ψ_α. We will return to $\hat{\Upsilon}$ in Section 1.11.[1]

1.9.3 Plane Waves

The fine structure constant should be a characteristic value associated with the unit cell. Per Section 1.7.3, plane wave states bounded in a finite region are written as

$$\phi_j(\mathbf{r}) = \frac{e^{i\mathbf{k}_j\cdot\mathbf{r}}}{\sqrt{V}} \quad , \tag{1.9.9}$$

where $\mathbf{k}_j$ is the j^{th} quantized wavenumber allowed by the finite boundary conditions. Although we expect $(\Phi\pi)^3$ to come from a third derivative, the V dependence in ϕ_j gives a hint of what is needed for α_{MCM}. $(\Phi\pi)^3$ is the volume of a 3D box whose sides have length $\Phi\pi$. The volume interpretation is interesting and deserving of further study because $\hat{\pi}$ and $\hat{\Phi}$ are associated with the $\mathcal{H}$ and Ω bounding branes of Σ^+ while the $\mathcal{A}$ and $\mathcal{H}$ bounding branes of Σ^- are associated with $\hat{2}$ and $\hat{\pi}$. This association of 2 and π in Σ^-, and Φ and π in Σ^+ is oddly similar to the arrangement of numbers in $2\pi+(\Phi\pi)^3$. However, the association of V with α_{MCM} does not directly relate to an operator eigenvalue $\hat{\alpha}|\Psi_\alpha\rangle = \alpha^{-1}|\Psi_\alpha\rangle$, apparently. Instead, the volume would show up in the allowed quantized $\mathbf{k}_j$ associated with ∂_x acting on ϕ_j. The discrete $\mathbf{k}_j$ and ω_j are

$$\mathbf{k}_j = 2\pi\left(\frac{j_x}{L_x}, \frac{j_y}{L_y}, \frac{j_z}{L_z}\right) \quad , \quad \text{and} \qquad \omega_j = \frac{k_j^2}{2\hbar\mu} \quad , \tag{1.9.10}$$

and we might expect some quantized spectrum for β in the MCM ansatz

$$\psi_j(\mathbf{x}, t, \chi^4) = \exp\big\{i\big(\mathbf{k}_j\cdot\mathbf{x} - \omega_j t + \beta_j\chi^4\big)\big\} \quad , \tag{1.9.11}$$

where j becomes a tuple of five integers. Quantization in β would follow from the unit cell's boundary conditions along the χ^4 direction.

Considering (1.9.11), spatial derivatives hitting $\mathbf{x}$ will produce a sum of three analytical terms not compatible with α_{MCM}. The original use case for ∂_x in [30] relied on a reduction to one spatial dimension (Appendix A) but the gradient acting on a 3D spatial wavefunction will return three summed factors of 2π, two of which do not

[1] A Schrödinger equation for the identity operator was an idea for the origin of α which was omitted from this book because it could not be quickly developed. However, one might attempt to write a Schrödinger equation for the identity operator to characterize changing scale from one brane to the next. This exercise would be guided by the intention to associate α_{MCM} with the changing level of aleph.

appear in α^{-1}_{MCM}. This suggests that $\hat{\alpha}$ should be a combination of ∂_0 and ∂_4, or a combination of $\{\partial_+, \partial_\varnothing, \partial_-\}$ derivatives. Allowable forms for $\hat{\alpha}|\Psi_\alpha\rangle$ include

$$\hat{\alpha}\big|\Psi_\alpha\big\rangle = i\big(\partial_0 + \partial_4\big)e^{i(\mathbf{k}_j\cdot\mathbf{x}-2\pi t-\Phi\pi\chi^4)} = 2\pi + \big(\Phi\pi\big)^3 \quad , \tag{1.9.12}$$

and

$$\hat{\alpha}\big|\Psi_\alpha\big\rangle = i\big(\partial_- - \partial_+^3\big)e^{i(\mathbf{k}_j\cdot\mathbf{x}-\omega_j t-2\pi\chi^4_- +\Phi\pi\chi^4_+)} = 2\pi + \big(\Phi\pi\big)^3 \quad . \tag{1.9.13}$$

Following the program of the particle in a box developed in [3, 30], one would determine which geometries are consistent with a given quantization for ω and β. However, a problem which remains will be an unbounded quantization spectrum leading to an infinite tier of eigenvalues for α. Experiment does not suggest that the FSC is only one dimensionless number from a large catalog of such numbers. At best, $\alpha_{\text{MCM}} \approx \alpha_{\text{MCM}}$ is one of three or four dimensionless coupling constants, the others being α_{Weak}, α_{Strong}, and possibly the numerically disparate α_{Grav} .

1.9.4 Disagreement Between α_{MCM} and α_{QED}

The L3 Collaboration writes the following [87].

> "At zero momentum transfer, the QED fine structure constant $\alpha(0)$ is very accurately known from the measurement of the anomalous magnetic moment of the electron and from solid-state physics measurements:
>
> $$\alpha^{-1}(0) = 137.03599976(50) \quad . \tag{1.9.14}$$
>
> In QED, vacuum polarization corrections to processes involving the exchange of virtual photons result in a Q^2 dependence, or running, of the *effective* fine-structure constant, $\alpha(Q^2)$."

Figure 14 shows that α^{-1}_{QED} tends to decrease with increasing energy so it is notable that

$$\alpha^{-1}_{\text{MCM}} = 2\pi + \big(\Phi\pi\big)^3 \approx 137.62788 \quad , \tag{1.9.15}$$

is higher even than what the L3 Collaboration have called $\alpha^{-1}(0)$:

$$\alpha^{-1}(0) - \alpha^{-1}_{\text{MCM}} \approx -0.59 \quad . \tag{1.9.16}$$

Therefore, it must be noted that the energy of scale of a process is not absolute. It depends on the renormalization scheme as well as the manner of association between Q^2 and the Mandelstam variables, as in Figure 15. It is a common convention to set

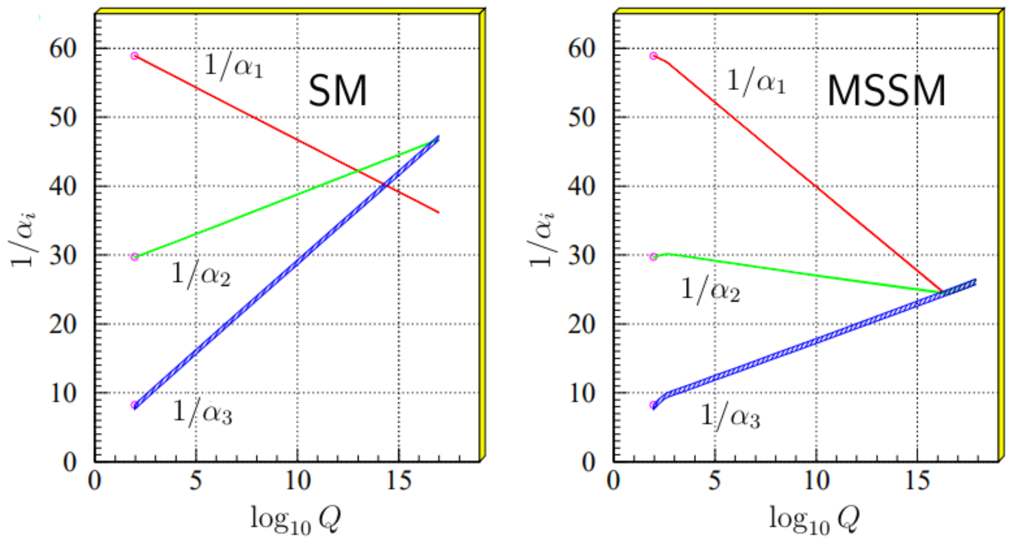

Figure 14: This figure shows an updated plot of Amaldi, de Boer, and Fürstenau [88]. In units natural to high energy physics, the α_i plotted here are the electromagnetic, weak, and strong coupling constants respectively. The standard model (left) nearly unifies the coupling constants of the forces but the (minimal) supersymmetric standard model (right) exactly unifies them at a given energy scale. These famous plots refute any detractors' claims about indisputable precision in the currently accepted value of α_{QED} ruling out a physical basis for α_{MCM}.

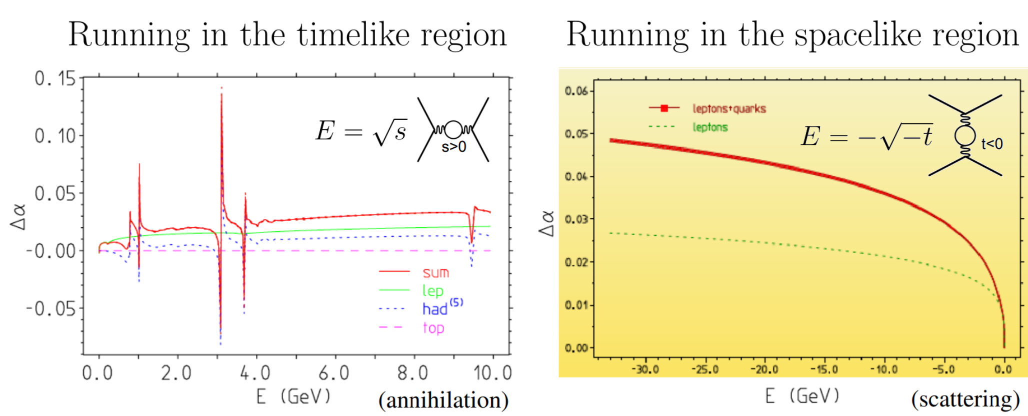

Figure 15: Plots of $\Delta\alpha$ vs E are taken from a seminar of Venanzoni [91] regarding results from the KLOE collaboration [90]. The s and t variables are the usual Mandelstam variables for particle scattering. With time increasing to the right in the inset particle diagrams, one observes that the exchanged photon moves through the timelike and spacelike regions in the respective interactions.

the energy scale of α_{QED} to the rest energy of the electron $E_e = 511\text{keV}$ so that

$$\alpha_{\text{QED}} \equiv \alpha(0) \quad \longrightarrow \quad \alpha_{\text{QED}} \equiv \alpha(E_e^2) = \frac{e^2}{4\pi\varepsilon_0\hbar c} \quad . \tag{1.9.17}$$

This allows us to suppose that $\alpha_{\text{MCM}}^{-1} > \alpha_{\text{QED}}^{-1}$ might be the true $\alpha^{-1}(0)$. Though the small scale of keV relative to the GeV scale in Figure 14 suggest that α_{MCM} probably does not lie on the linear trend of the standard model, the kinks at low energy the end of the supersymmetric standard model suggest that the uncharted low energy region might accommodate α_{MCM} as a value not on the trend line. Since the units of Figure 14 are not ones in which $\alpha_{\text{QED}}^{-1} \approx 137$, a calculation is required to determine whether even that constant lies on the trend line.

Several measurements of the running of α suggest that α_{MCM} is a reasonable number. The main results of the L3 collaboration reported in [87] were

$$\begin{aligned} \alpha^{-1}(-2.1) - \alpha^{-1}(-6.25) &= 0.78 \pm 0.26 \qquad \big(\Delta Q^2 = 4.15\big) \\ \alpha^{-1}(-12.25) - \alpha^{-1}(-3435) &= 3.80 \pm 1.29 \qquad \big(\Delta Q^2 = 3422\big) \quad , \end{aligned} \tag{1.9.18}$$

where Q^2 is in units of GeV^2. Referring to (1.9.16), one notes that

$$0.59 < 0.78 < 3.80 \quad . \tag{1.9.19}$$

This suggests the Q^2 difference between α_{MCM} and α_{QED} is less than 4GeV^2. This agrees with the supposition for $\alpha_{\text{MCM}} \equiv \alpha(0)$ and $\alpha_{\text{QED}} \equiv \alpha(E_e^2)$, if it isn't a bit larger than would be expected. The result reported by the OPAL collaboration in [89] was

$$\Delta\alpha(-6.07) - \Delta\alpha(-1.81) \approx 0.0044 \qquad \big(\Delta Q^2 = 4.26\big) \quad . \tag{1.9.20}$$

Using

$$\alpha(Q^2) = \frac{\alpha(0)}{1 - \Delta\alpha(Q^2)} \quad , \tag{1.9.21}$$

(given in [87]) to compute

$$\Delta\alpha_{\text{QED}} = 1 - \frac{\alpha_{\text{MCM}}}{\alpha_{\text{QED}}} \approx 0.0043 \quad , \qquad \text{and} \qquad \Delta\alpha_{\text{MCM}} = 0 \quad , \tag{1.9.22}$$

we find

$$\Delta\alpha_{\text{QED}}^{-1} - \Delta\alpha_{\text{MCM}}^{-1} \approx 0.0043 \quad . \tag{1.9.23}$$

OPAL's result also fits the present picture of α_{MCM}. In [90], the KLOE collaboration reports a measurement in the low energy region omitted from Figure 14. They find

$$\left|\frac{\alpha(Q^2_{\text{avg}})}{\alpha(0)}\right|^2 \approx 1.029 \quad , \quad \text{for} \quad 0.605\text{GeV} \leq Q \leq 0.975\text{GeV} \quad .^1 \tag{1.9.24}$$

Comparing the present model, we find

$$\left|\frac{\alpha_{\text{QED}}}{\alpha_{\text{MCM}}}\right|^2 \equiv \left|\frac{\alpha(E_e^2)}{\alpha(0)}\right|^2 \approx \left|\frac{137.03600^{-1}}{137.62788^{-1}}\right|^2 = 1.009 \quad . \tag{1.9.25}$$

So, while the discrepancy between α_{MCM} and α_{QED} might have seemed high, KLOE reports much greater running in the low energy region than might be intuited from the results of the L3 and OPAL collaborations [87, 89], or from Figure 14 [88]. In addition to the mild kinks at the low energy range of the supersymmetric model and the wide running observed by KLOE, the sharp resonance structure observed for α running in the timelike region (Figure 15) might easily accommodate the present supposition for α_{MCM}.

The MCM value for the the fine structure constant is well within the experimental bounds. The fact that $\alpha^{-1}_{\text{MCM}} > \alpha^{-1}_{\text{QED}}$ is well fitting to the theme of the MCM. Since the running of the fine structure constant is associated with an *effective* charge on the electron due to screening by vacuum polarization, a hypothetical $\alpha^{-1}_{\text{MCM}} < \alpha^{-1}_{\text{QED}}$ would force us to associate α_{MCM} with some effective $\alpha(Q^2)$ not well suited to the desired absoluteness of a fundamentally *ontological* picture. As it is, however, $\alpha^{-1}_{\text{MCM}} > \alpha^{-1}_{\text{QED}}$ allows to choose $\alpha_{\text{MCM}} = \alpha(0)$ as the perfect, non-effective value that one might associate with an underlying geometric structure of reality.

Overall, the main purpose of this section has been to refute detractors' claims that high precision in the currently accepted value of α_{QED} categorically rules out a physical basis for α_{MCM}. To that end, the following relevant excerpts appear in a publication of NIST [92] and a publication of Fritzsch [93].

> "Indeed, due to e^+e^- and other vacuum polarization processes, at an energy corresponding to the mass of the W boson (approximately 81 GeV, equivalent to a distance of approximately 2×10^{-17}m), $\alpha(m_W)$ is approximately 1/128 compared with its zero-energy value of approximately 1/137. Thus the famous number 1/137 is not unique or especially fundamental."

> "[*A*]t energies which were reached by the LEP–Accelerator,[2] of the order of 200 GeV, the associated value of the finestructure constant is more than

[1] This value is averaged from Table 2 in [90].
[2] These results are found in [87].

10% higher than at low energy. In any case this signifies that one should not attach a specific fundamental meaning to the numerical value of the finestructure constant."

These sources state what all subject matter experts already knew: detractors' citations to the 0.4% FSC discrepancy as conclusive evidence of terminal wrongness are nothing but the libelous vomit of those who would prey on the non-expertise of certain third parties.[1]

1.9.5 Grand Unification

The variation of the fundamental coupling constants is the matter at the heart of the grand unification of fundamental forces which the MCM hopes to achieve, as in Figure 14. In addition to seeking unification of the coupling constants at a given energy scale, now we might explore cases for all three $\alpha_i(0)$ to fall out of the ontological numbers combined with the geometry of the unit cell. Even the fourth coupling constant for gravity which is omitted from grand unification due to its vastly disagreeable scale (the hierarchy problem) might now be studied as a characterization of the changing scale from one level of aleph to another.

1.10 Quantum Gravity

1.10.1 Einstein's Equation

There are a few equations which can be used to initiate the MCM route to Einstein's equation. The original route in [3] was as follows. Suppose that the third chronological time derivative of the ansatz

$$\psi(x,t,\chi^4) = \exp\big\{ i\big(kx - \omega t + \beta\chi^4\big)\big\} \quad , \tag{1.10.1}$$

is equal to the translation operator definition of $\hat{M}^3$ (Section 1.7.1):

$$\begin{aligned} \hat{M}^3\big|\psi;\hat{\pi}\big\rangle &= \hat{M}^3\big|\psi;\hat{\pi}\big\rangle \\ \partial_0^3\big|\psi;\hat{\pi}\big\rangle &= \hat{\mathcal{J}}_-\hat{\mathcal{J}}_\varnothing\hat{\mathcal{J}}_+\big|\psi;\hat{\pi}\big\rangle \\ \big(-i\omega\big)^3\big|\psi;\hat{\pi}\big\rangle &= 2\pi\Phi\big|\psi;\hat{\pi}\big\rangle \\ 8i\pi^3\nu^3\big|\psi;\hat{\pi}\big\rangle &= 2\pi\big|\psi;\hat{\pi}\big\rangle + 2\pi\varphi\big|\psi;\hat{\pi}\big\rangle \quad .^2 \end{aligned} \tag{1.10.2}$$

[1] An example of such third parties would be the ones for whom Ellis and You concocted their lie about "reasonable doubt" in [28].

The final line follows from $\Phi = 1 + \varphi$. In Section 1.10.3, we will give a new, better motivation for operating differently with $\hat{M}^3$ on the left and right sides of (1.10.2). The purpose here, however, is to present the mechanism as it appeared previously.

In earlier sections, we discussed two different cases of orthogonality for MCM states. First, wavefunctions in each unit cell might be orthogonal from those in other unit cells. This condition is written

$$\big\langle \psi; \hat{e}^m_\mu \big| \psi; \hat{e}^n_\nu \big\rangle = \delta_{mn} \|\hat{e}^m_\mu\| \|\hat{e}^n_\nu\| \quad , \tag{1.10.3}$$

where m, n refer to the level of aleph. The other picture of orthogonality set wavefunctions in each brane orthogonal from those in every other brane:

$$\big\langle \psi; \hat{e}^m_\mu \big| \psi; \hat{e}^n_\nu \big\rangle = \delta_{\mu\nu}\delta_{mn} \|\hat{e}^m_\mu\| \|\hat{e}^n_\nu\| \quad . \tag{1.10.4}$$

If we are to proceed as in [3], it is required that we adopt the former convention of (1.10.3). The variants of ψ located in the $\mathcal{A}$-, $\mathcal{H}$-, or Ω-branes of any one unit cell cannot be linearly independent from each other. Linear dependence allows us to proceed from (1.10.2) by inserting the identity and rearranging the hats:[1]

$$\begin{aligned}
8i\pi^3\nu^3\big|\psi;\hat{\pi}\big\rangle &= 2\pi\frac{\|\hat{\Phi}\|}{\|\hat{\Phi}\|}\big|\psi;\hat{\pi}\big\rangle + 2\pi\varphi\frac{\|\hat{2}\|}{\|\hat{2}\|}\big|\psi;\hat{\pi}\big\rangle \\
8i\pi^3\nu^3\big|\psi;\hat{\pi}\big\rangle &= \frac{2\pi\|\hat{\pi}\|}{\Phi}\big|\psi;\hat{\Phi}\big\rangle + \frac{2\pi\varphi\|\hat{\pi}\|}{2}\big|\psi;\hat{2}\big\rangle \\
8\pi i\Phi\nu^3\big|\psi;\hat{\pi}\big\rangle &= 2\big|\psi;\hat{\Phi}\big\rangle + \big|\psi;\hat{2}\big\rangle \quad .
\end{aligned} \tag{1.10.5}$$

Due to the constant 8π appearing at the end of (1.10.5), and due to that alone, the resultant expression was recognized to be in the form of Einstein's equation

$$8\pi T_{\mu\nu} = G_{\mu\nu} + g_{\mu\nu}\Lambda \quad . \tag{1.10.6}$$

(The overall research program leading to the final line of (1.10.5) is summarized in Section 1.10.7.) Recasting (1.10.5) as Einstein's equation requires the introduction of

[2] In [3], the convention was such that $\hat{M}^3|\psi;\hat{\pi}^1\rangle = i\pi\Phi^2|\psi;\hat{\pi}^2\rangle$ rather than the current convention for $\hat{M}^3|\psi;\hat{\pi}^0\rangle = 2\pi\Phi|\psi;\hat{\pi}^1\rangle$.

[1] This procedure for rearranging hats follows from (1.2.22) in Section 1.2.1.

new variables:

$$\begin{aligned} i\Phi\nu^3\big|\psi;\hat{\pi}\big\rangle &\to T_{\mu\nu} \\ 2\big|\psi;\hat{\Phi}\big\rangle &\to G_{\mu\nu} \\ \big|\psi;\hat{2}\big\rangle &\to g_{\mu\nu}\Lambda \quad . \end{aligned} \tag{1.10.7}$$

Substitution of these variables back into the final line of (1.10.5) yields (1.10.6).

The interpretation of this result is that general relativity describes a condition in which the present is the sum of the past and the future. Intuitively, we have the stress-energy tensor $T_{\mu\nu}$ associated with the $\mathcal{H}$-brane. Less intuitively, the Einstein tensor $G_{\mu\nu}$ is associated with Ω and the cosmological constant is attached to $\mathcal{A}$. The meaning of $T_{\mu\nu} \in \mathcal{H}$ is clear enough but the meanings of the other assignments are not obvious. Furthermore, we have arbitrarily chosen the entire Einstein tensor

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{R}{2} g_{\mu\nu} \quad , \tag{1.10.8}$$

for association with Ω when we might have let $\Lambda \to 0$ and assigned the Ricci tensor and the Ricci scalar to Ω and $\mathcal{A}$ through maps other than those in (1.10.7). We have also assumed that the new variables are in one-to-one correspondence with the states rather than with their linear combinations.

The desire to phrase general relativity as a statement of the present being equal to a sum a contributions from the past and future is confounded (or complicated) when we include an iterator for the level of aleph:

$$\hat{M}^3|\psi;\hat{\pi}\rangle = 2\pi\Phi|\psi;\hat{\pi}\rangle \quad \longrightarrow \quad \hat{M}^3|\psi;\hat{\pi}^k\rangle = 2\pi\Phi|\psi;\hat{\pi}^{k+1}\rangle \quad . \tag{1.10.9}$$

With the k iterators, Einstein's equation tells us that the present on one level of aleph is equal to a sum of contributions from the past and future relative to some time in the future. Unfortunately, this interpretation is much less clean than what can be said in the absence of the iterators. To make better sense of the place for Einstein's equation, we must first refer to the picture of MCM cosmology states which have gone untreated thus far.

1.10.2 MCM Cosmology States

Even before the MCM particle scheme was introduced to solve the fundamental problem of QFT (Section 0.3) [6], the universe was treated as a quantum particle to resolve another question about why matter dominates over anti-matter in the cosmos [31]. Upon introducing a reverse time universe in fulfillment of a requirement for conserved momentum at a big bang (or big bounce), an eigenbasis of **quantum cosmology states** was defined for a time arrow operator [39]:

$$\begin{aligned}\hat{T}\big|t_+\big\rangle &= \big|t_+\big\rangle \\ \hat{T}\big|\text{bounce}\big\rangle &= 0 \\ \hat{T}\big|t_-\big\rangle &= -\big|t_-\big\rangle \quad .\end{aligned} \tag{1.10.10}$$

$|t_+\rangle$ is the state of a universe U_+ whose time arrow is such that the energy of that universe is positive-definite. $|t_-\rangle$ is the state of U_- whose energy is negative-definite. $|\text{bounce}\rangle$ is the state of $U_\pm$ simultaneously collapsed to a singularity.[1] An observer's inability to distinguish $|t_\pm\rangle$ led to the $|t_\star\rangle$ superposition as the observer's present moment. In quantum theory, ignorance about eigenstates is represented with superpositions of eigenstates, e.g.: Schrödinger's cat. The observer writes

$$\big|t_\star\big\rangle = \big|t_+\big\rangle + \big|t_-\big\rangle \quad , \tag{1.10.11}$$

when he is unable to determine if his present moment belongs to a positively or negatively increasing continuum of time. The $|t_\star\rangle = |t_+\rangle + |t_-\rangle$ relationship is important for MCM electrogravity (Section 18) and the connection of the unit cell to KKT (Section 17) [7]. A simple statement of what it means for the present to be defined as the sum of components from the past and future is found in the definition

$$A^\mu = \frac{1}{2}A^+_\mu + \frac{1}{2}A^-_\mu \quad , \tag{1.10.12}$$

which says that the EM potential 4-vector A_μ in $\mathcal{H}$ is defined by $A^\pm_\mu$ in $\Sigma^\pm$ (Section 16) [7]. The metric in $\mathcal{H}$ defined as a superposition of $g^\pm_{AB}$ as $\chi^4_\pm \to 0$ is another example. The exact details of these dependencies remain to be worked out but we have clarified what it means for the present to be defined as a sum of contributions from the past and future.

The MCM supposes that a cosmogenesis bounce event is equivalent to spontaneous pair creation in the quantum vacuum.[2] In [39], the $\widehat{\text{MCM}}$ operator was introduced

[1] In the MCM's original big bounce treatment [31, 39], $|\text{bounce}\rangle$ referred to the apex of a "quantum geometric bounce" rather than a true singularity. However, the language of quantum geometry has since been deprecated in the MCM. The original formulation made an appeal to Ashtekar's "repulsive force of quantum geometry" [57] to avoid total topological collapse at the bounce but it is likely that Ashtekar's avoidance of total collapse was only an artifact of his numerical algorithms, in the opinion of this writer. Presently, we *do* associate $\varnothing$ with a topological singularity of infinite curvature.

[2] A pair of universes coming into existence spontaneously is like pair creation in the vacuum while the total bounce process for a crunch followed by a bang is like annihilation to a photon followed by $\gamma \to e + p$.

to affect this pair creation as

$$\widehat{\mathrm{MCM}}\big|\mathrm{bounce}\big\rangle = \big|t_+\big\rangle + \big|t_-\big\rangle \ .^1 \tag{1.10.13}$$

The fourth time state $|t_\star\rangle$ is like a state in a Hilbert space of states at time t_0 corresponding to the present. Due to certain likenesses between the singular present moment and the singular apex of a big bounce, the present was identified with the bounce as

$$\big|t_\star\big\rangle \equiv \big|\mathrm{bounce}\big\rangle \qquad \Longrightarrow \qquad \hat{T}\big|t_\star\big\rangle = 0 \ . \tag{1.10.14}$$

Thus, the convergence of $U_\pm$ on the bounce was associated with the convergence of the past and future on the present. Subsequent work now suggests that the $|t_\star\rangle$ and $|\mathrm{bounce}\rangle$ cosmology states should be associated with the $\mathcal{H}$- and $\varnothing$-branes respectively. The $|t_\star\rangle \neq |\mathrm{bounce}\rangle$ structure is preferable for a number of reasons. For one, it avoids an implied identification between a spatially extended present moment with a non-extended big crunch singularity. Whether or not we identify the bounce with the present, we must reconcile

$$\big|t_\star\big\rangle = \big|t_+\big\rangle + \big|t_-\big\rangle \ , \quad \text{and} \qquad \widehat{\mathrm{MCM}}\big|t_\star\big\rangle = \big|t_+\big\rangle + \big|t_-\big\rangle \tag{1.10.15}$$

where the latter mimics (1.10.13) to say that we must be able to obtain past and future states from a state in the present. The equations in (1.10.15) can only be consistent if $\widehat{\mathrm{MCM}}$ is the identity operator, which is not *exactly* the intended meaning. In this section, we will show that $\widehat{\mathrm{MCM}}$ is a new completeness relation. Such relations are inserted into expressions as the identity.

The $|t_\pm\rangle$ states were originally associated with the positive and negative x^0 modes needed to conserve momentum at the big bang but subsequent work allows us to associate them with $\chi^4_\pm$. Moving in that direction, we will define a separate chirological time arrow operator

$$\begin{aligned} \hat{\mathcal{T}}\big|\chi^4_+\big\rangle &= \big|\chi^4_+\big\rangle \\ \hat{\mathcal{T}}\big|\chi^4_\varnothing\big\rangle &= 0 \\ \hat{\mathcal{T}}\big|\chi^4_-\big\rangle &= -\big|\chi^4_-\big\rangle \ . \end{aligned} \tag{1.10.16}$$

with a complete set of eigenstates:

$$\mathbb{1} = \sum_k \big|\chi^4_k\big\rangle\big\langle\chi^4_k\big| \ , \quad \text{where} \qquad k \in \{+,\varnothing,-\} \ . \tag{1.10.17}$$

[1]The $\widehat{\mathrm{MCM}}$ operator was called $\widehat{LQC}$ in [39].

When $\hat{\mathcal{T}}$ replaces $\hat{T}$ as it appears in (1.10.10), and when we identify $|\text{bounce}\rangle \equiv |\chi^4_\varnothing\rangle \neq |x^0\rangle$, we avoid a degeneracy of the 0 eigenvalue between $|t_\star\rangle$ and $|\text{bounce}\rangle$. Instead, they are eigenstates of different operators reflecting different physical conditions. The chronological time cannot exist at all in a singularity such as $|\text{bounce}\rangle$ because time and space are condensed to a point. On the other hand, we are well motivated to have $\chi^4_\varnothing$ already defined at the singularity because it is an abstract coordinate. (Recall that the singularity is associated with the embedded physical metric, not the 5D metric.) The $\hat{T}$ time arrow states may be connected to the ontological states as

$$\left.\begin{aligned} \big|t_+\big\rangle \equiv \big|x^0_+\big\rangle = \big|\psi;\hat{\Phi}\big\rangle = \big|\psi;\Omega\big\rangle \\ \big|t_\star\big\rangle \equiv \big|x^0\big\rangle = \big|\psi;\hat{\pi}\big\rangle = \big|\psi;\mathcal{H}\big\rangle \\ \big|t_-\big\rangle \equiv \big|x^0_-\big\rangle = \big|\psi;\hat{2}\big\rangle = \big|\psi;\mathcal{A}\big\rangle \end{aligned}\right\} \quad\Longrightarrow\quad \begin{aligned} \hat{T}\big|\psi;\hat{\Phi}\big\rangle &= \big|\psi;\hat{\Phi}\big\rangle \\ \hat{T}\big|\psi;\hat{\pi}\big\rangle &= 0 \\ \hat{T}\big|\psi;\hat{2}\big\rangle &= -\big|\psi;\hat{2}\big\rangle \quad , \end{aligned} \tag{1.10.18}$$

but it remains to be determined if $t_\pm$ are the past and future of $x^0 \in \mathcal{H}$, if they are $x^0_+ \in \Omega$ and $x^0_- \in \mathcal{A}$, or if these possibilities are the same. If we use the ontological basis conventions in (1.10.18), the chirological states must have some other identities:

$$\left.\begin{aligned} \big|t_+\big\rangle \equiv \big|\chi^4_+\big\rangle = \big|\psi;\Sigma^+\big\rangle \\ \big|\text{bounce}\big\rangle \equiv \big|\chi^4_\varnothing\big\rangle = \big|\psi;\varnothing\big\rangle \\ \big|t_-\big\rangle \equiv \big|\chi^4_-\big\rangle = \big|\psi;\Sigma^-\big\rangle \end{aligned}\right\} \quad\Longrightarrow\quad \begin{aligned} \hat{\mathcal{T}}\big|\psi;\Sigma^+\big\rangle &= \big|\psi;\Sigma^+\big\rangle \\ \hat{\mathcal{T}}\big|\psi;\varnothing\big\rangle &= 0 \\ \hat{\mathcal{T}}\big|\psi;\Sigma^-\big\rangle &= -\big|\psi;\Sigma^-\big\rangle \quad . \end{aligned} \tag{1.10.19}$$

As a guess for how we might describe the new chirological states with the ontological basis, we will introduce notation such that

$$\begin{aligned} \big|\psi;\Omega\big\rangle &= \big|\psi(x);\hat{\Phi}\big\rangle & \qquad \big|\psi;\Sigma^+\big\rangle &= \big|\psi(\chi);\hat{\Phi}\big\rangle \\ \big|\psi;\mathcal{H}\big\rangle &= \big|\psi(x);\hat{\pi}\big\rangle & \qquad \big|\psi;\varnothing\big\rangle &= \big|\psi(\chi);\hat{\imath}\big\rangle \\ \big|\psi;\mathcal{A}\big\rangle &= \big|\psi(x);\hat{2}\big\rangle & \qquad \big|\psi;\Sigma^-\big\rangle &= \big|\psi(\chi);\hat{2}\big\rangle \quad . \end{aligned} \tag{1.10.20}$$

This notation is made clearer when the $\psi(\chi)$ wavefunction is renamed with the letter ξ, e.g.:

$$\big|\xi;\Sigma^-\big\rangle = \big|\xi(\chi);\hat{2}\big\rangle \quad , \qquad \text{or} \qquad \big|\xi;\Sigma^-\big\rangle = \big|\xi;\hat{2}\big\rangle \quad . \tag{1.10.21}$$

We will use the ξ notation in following sections but presently we will continue with the $|x^0\rangle$ and $|\chi^4\rangle$ notations.

The structure of quantum theory is such that there should exist a transformation matrix for expressing an arbitrary cosmology state in the chronological or chirological time arrow eigenbasis. If $\widehat{\text{MCM}}$ is the completeness relation, the $|t_\star\rangle = |x^0\rangle$ state is written in the chirological basis as

$$\big|x^0\big\rangle = \widehat{\text{MCM}}\big|x^0\big\rangle = \sum_k \big|\chi^4_k\big\rangle\big\langle\chi^4_k\big|x^0\big\rangle = \sum_k c_k\big|\chi^4_k\big\rangle \quad , \tag{1.10.22}$$

where $k \in \{+, \varnothing, -\}$. Letting $\widehat{\text{MCM}}$ be the completeness relation for chronological states and $k \in \{+, \star, -\}$, we obtain

$$\big|\chi^4_\varnothing\big\rangle = \widehat{\text{MCM}}\big|\chi^4_\varnothing\big\rangle = \sum_k \big|t_k\big\rangle\big\langle t_k\big|\chi^4_\varnothing\big\rangle = \sum_k c_k\big|t_k\big\rangle \quad , \tag{1.10.23}$$

By setting $c_\star = c_\varnothing = 0$, we will obtain equations roughly in the form of (1.10.13):

$$\big|x^0\big\rangle = \big|\chi^4_+\big\rangle + \big|\chi^4_-\big\rangle \quad , \quad \text{and} \qquad \big|\chi^4_\varnothing\big\rangle = \big|t_+\big\rangle + \big|t_-\big\rangle \quad . \tag{1.10.24}$$

However, this does not necessarily reflect the argument that an observer's inability to distinguish $t_\pm$ should lead to the superposition

$$\big|t_\star\big\rangle = \big|t_+\big\rangle + \big|t_-\big\rangle \quad . \tag{1.10.25}$$

As a guiding principle, one notes that the orthogonal eigenvectors of an operator can never be expressed as linear combinations of the other eigenvectors. This is contrary to what is presumed for the $|x^0\rangle = |x^0_+\rangle + |x^0_-\rangle$ relationship if $|x^0\rangle$ is an eigenvector of $\hat{T}$ with eigenvalue 0. The similar $|x^0\rangle = |\chi^4_+\rangle + |\chi^4_-\rangle$ has no such problem so $|x^0\rangle \neq |\text{bounce}\rangle$ is implied. We will revisit these issues in Section 12 when presenting time arrow spinor states that only have ± 1 eigenvalues. Presently, more thinking is required to understand what reason we might have to set $c_\star = c_\varnothing = 0$ in the expansions of $|t_\star\rangle$ and $|\text{bounce}\rangle$, or if we should work in a basis that does not have a zero eigenvalue. For example, it was decided in [39] that we should write $\widehat{\text{MCM}}|\text{bounce}\rangle = |t_+\rangle + |t_-\rangle$ but not

$$\widehat{\text{MCM}}\big|\text{bounce}\big\rangle = \big|t_+\big\rangle + \big|t_-\big\rangle + \big|t_\star\big\rangle \quad , \tag{1.10.26}$$

because $|t_\star\rangle$ and $|\text{bounce}\rangle$ were identified. Subsequently, we have disassociated them as $\mathcal{H}$ and $\varnothing$ so we must consider (1.10.26) as a valid case of (1.10.23) deserving further inquiry with $c_\star \neq 0$.

Now that we have introduced **time arrow eigenstates**, we have set the stage for a new approach to quantum gravity. Then we will return to MCM cosmology states in Section 12.

1.10.3 A New Approach to Quantum Gravity

The completeness relations for time arrow states allow us to more tidily phrase general relativity as a relationship between the stress-energy tensor in the present and a sum of contributions from the past and future. In the method of Section 1.10.1, we supposed that there should be two different ways for $\hat{M}^3$ to act on ψ but this was not well motivated. To proceed with a better derivation of Einstein's equation, we will operate with $\hat{M}^3$ on a state written in the chronological and chirological eigenbases. Using the ξ notation as in (1.10.21), completeness yields

$$\big|\psi;\hat{\pi}\big\rangle = \sum_k \big|\xi;\hat{e}_k\big\rangle\big\langle\xi;\hat{e}_k\big|\psi;\hat{\pi}\big\rangle = c_+\big|\xi;\hat{\Phi}\big\rangle + c_\varnothing\big|\xi;\hat{i}\big\rangle + c_-\big|\xi;\hat{2}\big\rangle \quad . \tag{1.10.27}$$

Acting with $\hat{M}^3$ on both sides yields

$$\hat{M}^3\big|\psi;\hat{\pi}\big\rangle = \hat{M}^3\Big(c_+\big|\xi;\hat{\Phi}\big\rangle + c_\varnothing\big|\xi;\hat{i}\big\rangle + c_-\big|\xi;\hat{2}\big\rangle\Big) \quad . \tag{1.10.28}$$

Now we may say that $\hat{M}^3$ has different representations when it acts on states written in the different time arrow eigenbases. For example, the $\hat{S}_z$ spin operator is only $\hbar/2$ times the σ_z Pauli matrix when it operates on states written in the S_z eigenbasis. It takes a different form when it operates on states written in the S_x or S_y eigenbases. One of the major deficiencies in the MCM has been the lack of a good reason for why $\hat{M}^3$ might act on a state in two different ways and now we have one: $\hat{M}^3$ acts differently on chronological states than chirological ones. The extent to which such a mechanism was a missing puzzle piece in the MCM cannot be overstated. (1.10.28) is probably the most significant new result reported in this book.

To proceed from (1.10.28) differently than the previous derivation starting at (1.10.2), we will work in the picture where states in different branes are linearly independent:

$$\big\langle\psi;\hat{e}^m_\mu\big|\psi;\hat{e}^n_\nu\big\rangle = \delta_{mn}\delta_{\mu\nu}\|\hat{e}^m_\mu\|\|\hat{e}^n_\nu\| \quad .^1 \tag{1.10.29}$$

Once again assuming that $\hat{M}^3$ operates on the chronological state as the third chronological time derivative, we have

$$8i\pi^3\nu^3\big|\psi;\hat{\pi}\big\rangle = \hat{M}^3\Big(c_+\big|\xi;\hat{\Phi}\big\rangle + c_\varnothing\big|\xi;\hat{i}\big\rangle + c_-\big|\xi;\hat{2}\big\rangle\Big) \quad . \tag{1.10.30}$$

[1] Compare to (1.10.3).

Assuming $c_\varnothing = 0$, this reduces to Einstein's equation via

$$\begin{aligned} i\pi^2\nu^3\big|\psi;\hat{\pi}\big\rangle &\to T_{\mu\nu} \\ c_+\hat{M}^3\big|\xi;\hat{\Phi}\big\rangle &\to G_{\mu\nu} \\ c_-\hat{M}^3\big|\xi;\hat{2}\big\rangle &\to g_{\mu\nu}\Lambda \quad , \end{aligned} \tag{1.10.31}$$

or similar. The cases for $c_\varnothing \neq 0$ would be accommodated by the parts of $G_{\mu\nu} = R_{\mu\nu} - \frac{R}{2}g_{\mu\nu}$. Since we have taken the states in different branes to be linearly independent, we might write the $c_\varnothing = 0$ case as

$$\begin{aligned} i\pi^2\nu^3\big|\psi;\hat{\pi}\big\rangle &\to T_{\mu\nu} \\ c_+\hat{M}^3\big|\xi;\hat{\Phi}\big\rangle &\to R_{\mu\nu} \\ c_-\hat{M}^3\big|\xi;\hat{2}\big\rangle &\to \left(\Lambda - \frac{R}{2}\right) g_{\mu\nu} \quad . \end{aligned} \tag{1.10.32}$$

The previous definition for the new variables (Section 1.10.1)

$$\begin{aligned} i\Phi\nu^3\big|\psi;\hat{\pi}\big\rangle &\to T_{\mu\nu} \\ 2\big|\psi;\hat{\Phi}\big\rangle &\to G_{\mu\nu} \\ \big|\psi;\hat{2}\big\rangle &\to g_{\mu\nu}\Lambda \quad . \end{aligned} \tag{1.10.33}$$

left an open question about how the same ψ could be mapped to three different tensors when the $\hat{e}_\mu$ do not analytically represent much more than a change of scale. In (1.10.31) and (1.10.32), this problem may be avoided if chirological states are not eigenstates of $\hat{M}^3$. Work is needed to develop the $\psi(x)$ and $\xi(\chi)$ analytical representations of the time arrow states and to determine the transformation equations for obtaining the gravitational theory. Finding the exact correspondence between MCM states and GR tensors is the principal outstanding work unit for MCM quantum gravity.

Overall, the language of the respective time basis states answers a question which was left open in previous descriptions of MCM quantum gravity [1,3,71,94–96]. Now $\hat{M}^3$ can operate on the same state in two different ways if the state is represented in two different eigenbases. Finally, the identification of $\hat{M}^3$ as a third time derivative in one representation remains in good agreement with our other intention to generate the $(\Phi\pi)^3$ term needed for α_{MCM}.

1.10.4 Comparison to Higgs' Seminal Result

To demonstrate that the MCM mechanism for quantum gravity represents a standard method in physics, we will compare it to the method used by Higgs in his 1964 paper regarding what is now called the Higgs(–Englert–Brout–Guralnik–Hagen–Kibble) mechanism. Higgs wrote the following [12].

> "[*Consider the case in which two*] scalar fields φ_1, φ_2 and a real vector field A_μ interact through the Lagrangian density
>
> $$L = -\frac{1}{2}\big(\nabla\varphi_1\big)^2 - \frac{1}{2}\big(\nabla\varphi_2\big)^2 - V(\varphi_1^2+\varphi_2^2) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \quad , \tag{1.10.34}$$
>
> where
>
> $$\begin{aligned} \nabla_\mu\varphi_1 &= \partial_\mu\varphi_1 - eA_\mu\varphi_2 \\ \nabla_\mu\varphi_2 &= \partial_\mu\varphi_2 + eA_\mu\varphi_1 \\ F_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu \quad , \end{aligned} \tag{1.10.35}$$
>
> e is a dimensionless coupling constant, and the metric is taken as $-+++$. L is invariant under simultaneous gauge transformations of the first kind on $\varphi_1 \pm i\varphi_2$ and the second kind on A_μ. Let us suppose that $V'(\varphi_0^2) = 0, V'(\varphi_0^2) > 0$; then spontaneous breakdown of U(1) symmetry occurs. Consider the equations (derived from [*(1.10.34)*] by treating $\Delta\varphi_1, \Delta\varphi_2$, and A_μ as small quantities) governing the propagation of small oscillations about the 'vacuum' solutions $\varphi_1(x) = 0, \varphi_2(x) = \varphi_0$:
>
> $$\begin{aligned} \partial^\mu\big\{\partial_\mu\big(\Delta\varphi_1\big) - e\varphi_0 A_\mu\big\} &= 0 \quad , \\ \big\{\partial^2 - 4\varphi_0^2 V''(\varphi_0^2)\big\}\big(\Delta\varphi_2\big) &= 0 \quad , \\ \partial_\nu F^{\mu\nu} &= e\varphi_0\{\partial^\mu\big(\Delta\varphi_1\big) - e\varphi_0 A_\mu\} \quad . \end{aligned} \tag{1.10.36}$$
>
> Equation [*(1.10.36b)*] describes wave whose quanta have (bare) mass $2\varphi_0\{V''(\varphi_0^2)\}^{1/2}$; Eqs. [*(1.10.36a)*] and [*(1.10.36c)*] may be transformed, by the introduction of new variables
>
> $$\begin{aligned} B_\mu &= A_\mu - \big(e\varphi_0\big)^{-1}\partial_\mu\big(\Delta\varphi_1\big) \quad , \\ G_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu = F_{\mu\nu} \quad , \end{aligned} \tag{1.10.37}$$
>
> into the form
>
> $$\partial_\mu B^\mu = 0 \quad , \quad \partial_\nu G^{\mu\nu} + e^2\varphi_0^2 B^\mu = 0 \quad . \tag{1.10.38}$$

Equation [*(1.10.38)*] describes vector waves whose quanta have (bare) mass $e\varphi_0$. In the absence of the gauge field coupling ($e=0$) the situation is quite different: Equations [*(1.10.36a)*] and [*(1.10.36c)*] describe zero-mass scalar and vector bosons, respectively. In passing, we note that the right-hand side of [*(1.10.36c)*] is just the linear approximation to the conserved current[.]"

In (1.10.35), Higgs assumes his scalar fields φ_1, φ_2 obey certain equations. Similarly, we have assumed that there exist two different, complete time arrow eigenbases satisfying

$$\hat{M}^3\big|\psi;\hat{\pi}\big\rangle = \partial_0^3\big|\psi;\hat{\pi}\big\rangle \quad , \quad \text{and} \qquad \big|\psi;\hat{\pi}\big\rangle = c_+\big|\xi;\hat{\Phi}\big\rangle + c_\varnothing\big|\xi;\hat{\imath}\big\rangle + c_-\big|\xi;\hat{2}\big\rangle \quad . \tag{1.10.39}$$

Higgs imposes a broken U(1) symmetry by setting $V'(\varphi_0^2)=0$ and $V'(\varphi_0^2)>0$. We have set $c_\varnothing=0$ to write

$$8i\pi^3\nu^3\big|\psi;\hat{\pi}^1\big\rangle = c_+\hat{M}^3\big|\xi;\hat{\Phi}\big\rangle + c_-\hat{M}^3\big|\xi;\hat{2}\big\rangle \quad . \tag{1.10.40}$$

Next, Higgs introduces variables B_μ and $G_{\mu\nu}$. Our next step was to introduce new variables as

$$\begin{aligned} i\pi^2\nu^3\big|\psi;\hat{\pi}\big\rangle &\to T_{\mu\nu} \\ c_+\hat{M}^3\big|\xi;\hat{\Phi}\big\rangle &\to G_{\mu\nu} \quad . \\ c_-\hat{M}^3\big|\xi;\hat{2}\big\rangle &\to g_{\mu\nu}\Lambda \quad . \end{aligned} \tag{1.10.41}$$

Written in his new variables, Higgs claims that (1.10.38) "describes vector waves whose quanta have (bare) mass $e\varphi_0$." We have claimed that (1.10.40) written in terms of our new variables is Einstein's equation, which is true.

The main deficiency of the MCM program relative to Higgs' is that the new MCM variables are introduced by an unstated correspondence between rank-2 tensors whereas Higgs has given his new variables with definite tensorial equations. This deficiency requires remediation in future work.

1.10.5 An Alternative for $\hat{M}^3$

MCM quantum gravity is a relationship between $\hat{M}^3$ acting on a state's representations in the chronological and chirological bases. Therefore, one might ask if $\hat{M}^3 : \mathcal{H}\to\Omega\to\mathcal{A}\to\mathcal{H}$ can be achieved by acting on chirological states rather than the $|\psi;\hat{\pi}\rangle$ chronological state we have discussed. The representation of a chronological

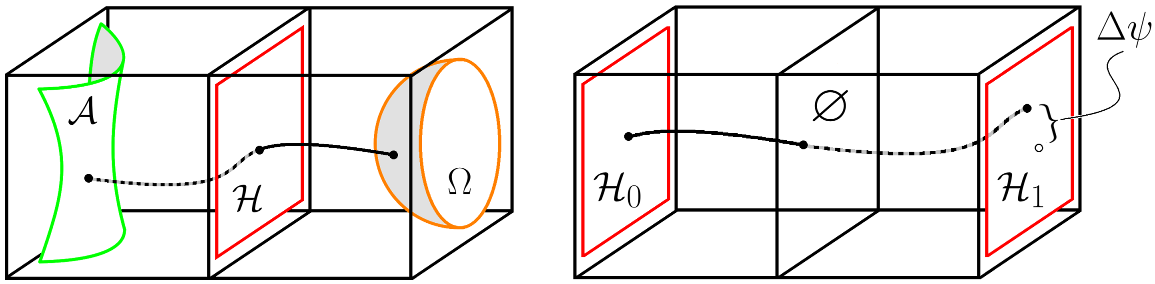

Figure 16: This figure illustrates a method in which one might avoid parameterizing a smooth curve through $\varnothing$. On the left, the solid and dashed lines show the trajectories of χ^4_+ and χ^4_- eigenstates away from $\mathcal{H}$ along their respective arrows of time. By concatenating the path through Σ^- to the end of the path through Σ^+, we obtain an evolved $\psi + \Delta\psi$ state in $\mathcal{H}_1$.

eigenstate in the chirological basis as

$$\big|\psi, t_0; \hat{\pi}^0\big\rangle \equiv \big|x^0\big\rangle = \big|\chi^4_+\big\rangle + \big|\chi^4_-\big\rangle \quad , \tag{1.10.42}$$

suggests that $\hat{M}^3$ acting on $|x^0\rangle$ would act on $|\chi^4_+\rangle$ and $|\chi^4_-\rangle$ simultaneously in the other representation. Recalling that the arrows of time point oppositely in $\Sigma^\pm$, $\hat{M}^3$ might evolve $|\chi^4_+\rangle$ and $|\chi^4_-\rangle$ to Ω and $\mathcal{A}$ such that the evolved state $|\psi, t_1; \hat{\pi}^1\rangle$ is determined from the difference or ratio across $\varnothing$ without computing a smooth trajectory through it. By fixing the path in Σ^+ and adjusting the path in Σ^- to fit a matching condition at $\varnothing$, the $|\psi, t_0; \hat{\pi}^0\rangle$ initial state would be adjusted to the $|\psi, t_1; \hat{\pi}^1\rangle$ final state in $\mathcal{H}_1$, as in Figure 16.

This method would be a trick for computing the steps of $\hat{M}^3$ out of order so as to avoid computing a step of smooth evolution through $\varnothing$. One would attempt to correlate the $\Delta\psi$ obtained from this method with the $\Delta\psi$ obtained from the Schrödinger equation. The conjecture that this parallel method for $\hat{M}^3$ might exist is included here in large part because the possibility for reverse engineering a solution from the Schrödinger equation represents a definite work unit with an absolute calculation as its starting point.

1.10.6 The Planck Law

One of the most exciting features of the MCM mechanism for quantum gravity is the exotic ν^3 frequency dependence in the stress-energy tensor:

$$i\pi^2\nu^3\big|\psi; \hat{\pi}\big\rangle \to T_{\mu\nu} \quad . \tag{1.10.43}$$

Physics' foremost setting for ν^3 is the Planck law

$$B(\nu,T)=\frac{2h\nu^3}{c^2}\frac{1}{e^{\frac{h\nu}{kT}}-1}\quad . \tag{1.10.44}$$

B is called the spectral radiance of blackbody radiation. The function $B(\nu,T)$ returns the energy carried by blackbody photons in each slice of constant wavelength at a given temperature. Similarly, we have obtained Einstein's equation by associating the $\nu^3\psi$ term with the stress-energy tensor in $\mathcal{H}$ which is the $\chi^4=0$ slice of the χ^4 spectrum (up to some nuance about $\chi^4_\pm=0$ not being defined.) This likeness of the stress-energy tensor and the spectral radiance is exciting because the ν^3 dependence is already known to describe energy per slice. The Planck law is approximately the only known place in physics where ν^3 appears. This congruence in the ν^3 dependency is interpreted as another strong hint that the MCM is producing results which deserve further study.

Written it terms of the wavelength, the Planck law is

$$B(\lambda,T)=-B(\nu,T)\frac{d\nu}{d\lambda}=\frac{2hc^2}{\lambda^5}\frac{1}{e^{\frac{hc}{\lambda kT}}-1}\quad . \tag{1.10.45}$$

This formula is true only for blackbody photons having dispersion relation $\omega(\lambda)=2\pi c\lambda^{-1}$. The antiderivatives of (1.10.44) and (1.10.45) are proportional to ν^4 and λ^{-4}, and it is the antiderivatives which obey the $\lambda\nu=c$ on-shell dispersion relation for photons. Namely, it is only in the integrated radiance that we may make direct substitutions with $\omega(\lambda)$. For example, if we plug the photonic dispersion relation into (1.10.43), we get

$$i\pi^2\nu^3\big|\psi;\hat{\pi}\big\rangle=\frac{i\pi^2c^3}{\lambda^3}\big|\psi;\hat{\pi}\big\rangle \tag{1.10.46}$$

which is not in the λ^{-5} form of (1.10.45). To preserve the relationship between the state corresponding to the stress-energy tensor and the Planck law, we must associate the integrated Planck law with the integrated wavefunction, as in the Dirac bra-ket. This is well reasoned because the wavefunction describing probability amplitude in some non-singular region of space would be associated with the Planck radiance in some non-singular band of the EM spectrum. Infinitesimal probability amplitude per position is matched with infinitesimal energy per wavelength. The expressions must be integrated for comparison to observables.

The Stefan–Boltzmann law says that the total emitted blackbody energy (per unit time) is equal to a constant times the fourth power of the temperature. Therefore, we would seek to associate the normalization of the MCM state with the temperature.

χ^4 describes the relative scale of the normalization of states among different branes so we may seek to associate χ^4 with the thermodynamic temperature T in $B(\nu, T)$ suggesting $(\nu, T) \to (x^0, \chi^4)$. With this identification of variables, it follows that the stress-energy tensor in question is the one at a definite chronological time in the brane whose scale is set by χ^4. In good agreement, the state $|\psi;\hat{\pi}\rangle$ which maps to $T_{\mu\nu}$ is implicitly $|\psi, t; \hat{\pi}\rangle$ at some definite time t. Furthermore, Wien's displacement law predicts the peak of the spectral intensity function and this should be associated with the expectation of some operator operating on the state associated with $T_{\mu\nu}$.

1.10.7 A Large Enough Number of Coincidences

If the proof is in the pudding, we have only presented an exceptional basket of ingredients for $\hat{M}^3$ and its use cases. If $\hat{M}^3$ should never pan out, other results regarding the Riemann hypothesis, classical electrogravity, and the fundamental problem of QFT will stand on their own. Experimental data will eventually confirm that the spectrum of MCM lattice vibrations is the true particle spectrum, or it will not. In the hope and belief that $\hat{M}^3$ will pan out, the purpose of this section is to review and summarize a large number of positive results following from $\hat{M}^3$ and leading to Einstein's equation. These results support the supposed existence of the new variables needed to obtain general relativity from a picture of quantum mechanics.

Firstly, it must be emphasized that the original discovery of Einstein's equation in the MCM was not goal-sought. When it was found, there was no intention to find it. It was not recognized until it had already been written. After discovering $2\pi + (\Phi\pi)^3 \approx 137$, the operator $\hat{M}^3$ *was* goal-sought toward $(\Phi\pi)^3$ but that was not the case for the dimensionless 8π in

$$8\pi T_{\mu\nu} = G_{\mu\nu} + g_{\mu\nu}\Lambda \quad . \tag{1.10.47}$$

Neither was it the intention to show that GR is a restatement of the $\widehat{\text{MCM}}|t_\star\rangle = |t_+\rangle + |t_-\rangle$ equation which had already been supposed as the MCM's philosophical kernel [31, 39]. When 8π first appeared in 2012 [3], the context had nothing to do with GR. Given a translation operator definition of $\hat{M}^3$ as in

$$\hat{M}^3\big|\psi;\hat{\pi}^0\big\rangle = 2\pi\Phi\big|\psi;\hat{\pi}^1\big\rangle \quad ,^1 \tag{1.10.48}$$

it was asked what would happen if $\hat{M}$ was a time derivative. The result which followed was described in Section 1.10.1. New variables were introduced and the result was Einstein's equation [3, 95].

[1] A different constant than $2\pi\Phi$ was used in [3].

With the serendipity of the development now emphasized, it is acknowledged that the number of hypothesized and/or supposed inputs required to construct the original mechanism [3, 95] was large enough to generate the superficial appearance that a sufficiently long string of suppositions can be used to output any desired result. However, the quantum gravity result, which is a new tool for synthesizing the objects of two disparate mathematical languages, was not desired. It fell out on its own from unrelated thinking. Ten years later, the result is now greatly improved, as in Section 1.10.3. We have reduced the number of unanswered questions left by the original derivation. Those questions included the following.

- Why should $\hat{M}^3$ act on ψ in two different ways?
- If $\hat{M}^3$ does act on ψ in two different ways, why should one of them be a third time derivative?
- Why should we invoke the given numerical values for the ontological basis at all when linearly independent bases are usually defined by orthogonality irrespective of magnitude?
- Even if the above are granted, what is the definite relationship between the $|\psi;\hat{e}_\mu\rangle$ states and the objects in Einstein's equation?

$\hat{M}^3$ should have a different representation when acting on the eigenstates of the chronological and chirological time arrow operators. This answers the first question about why the other questions are worth asking. The second question asks why $\hat{M}^3$ should take the form of the third time derivative needed to generate 8π and the ν^3 connection to Planck's law. This question remains open but $\hat{M}^3=\partial^3$ was already found to be useful for MCM work predating the quantum gravity application. It was expected that a third derivative is needed for $\hat{\alpha}|\Psi_\alpha\rangle=\alpha^{-1}_{\text{MCM}}|\Psi_\alpha\rangle$. The third derivative was also contextualized by the MCM reference to the theory of advanced and retarded EM potentials in [30]. This context predated the GR application, i.e.: the Abraham-Lorentz force

$$\mathbf{F}_{\text{AL}}=m(\ddot{\mathbf{x}}-\tau\dddot{\mathbf{x}}) \quad , \tag{1.10.49}$$

for radiation damping (Section 16) brought in a third time derivative a year before Einstein's equation was obtained. Finally, when it was observed that Laithwaite had suggested the time derivative of acceleration—another third derivative—as a possible cause for the anti-gravity effects observed in spinning discs [97, 98], this writer was inspired to explore the ansatz for $\hat{M}^3\propto\partial_t^3$. Einstein's equation was derived forthwith [3].

Another unanswered question regards the number-theoretical assignments for the ontological basis. The best that can be said is that the $\{\hat{e}_{\mathcal{A}}, \hat{e}_{\mathcal{H}}, \hat{e}_{\Omega}\}$ set of basis vectors, like the proposal to use the ∂_t^3 operator, was already entertained independently for reasons unrelated to quantum gravity [30]. Only later were the ontological numbers found to output Einstein's equation [3].

When the dimensionless coefficient 8π familiar from $8\pi T_{\mu\nu} = G_{\mu\nu} + g_{\mu\nu}\Lambda$ appeared, it appeared on the heels of another famous dimensionless constant: α_{MCM} [3, 30]. Furthermore, the emergence of Einstein's equation as a formal restatement of the $|t_\star\rangle = |t_+\rangle + |t_-\rangle$ idea at the heart of the MCM was too much to be assigned as mere coincidence in the eyes of this writer. Writing $\widehat{\text{MCM}}|\text{bounce}\rangle = |t_+\rangle + |t_-\rangle$ as Einstein's equation makes the tantalizing suggestion that the MCM requirement for global conservation of cosmological momentum is a restatement of the law already recorded in GR. 8π following so closely after α_{MCM} may be written off as mere coincidence by third parties but, as the personal pet project of this writer, the coincidence hypothesis is rejected on the basis of too much coincidence. To argue that the reader should also see more coincidence than should be ignored, we will briefly resummarize the evolution of ideas.

The Theory of Negative Time A thermodynamic paradox arises in closed universe models when singularities at past and future timelike infinity are identical. The second law of thermodynamics requires that the state at future timelike infinity should have much higher entropy than the state at past timelike infinity. This is resolved by the introduction of two universes coevolving simultaneously with opposite arrows of time [31]. The total entropy of both universes is a constant when the entropic increase of one is offset by the decrease in the other. As the log of the number of microstates, the entropy should not be affected by any scale factor. Later, it was determined that a universe with a reversed time arrow is also required to fix a problem of non-conserved momentum in big bang models [39]. The big bang should decay to a superposition of positive and negative time modes:

$$\widehat{\text{MCM}}\big|\text{bounce}\big\rangle = \big|t_+\big\rangle + \big|t_-\big\rangle \quad . \tag{1.10.50}$$

A Context for Retrocausality Transport of an observer's inertial frame though the bounce requires that the present moment should also be resolved in positive and negative time modes. In other words, if an observer in the big bang sees a superposition of two opposite time arrow states converging on his position, then he should see that at any other time as well. Hence, we arrive at

$$\widehat{\text{MCM}}\big|t_\star\big\rangle = \big|t_+\big\rangle + \big|t_-\big\rangle \quad , \tag{1.10.51}$$

Fixation on the bounce as a novel moment in [31] was supplanted by the present being taken as the novel moment of greatest interest [39]. This paved the way for the connection to quantum mechanical Hilbert spaces of states at the present time.

(1.10.51) suggests equal places for causality and retrocausality. The main venue for such mechanisms in physics is the theory of the advanced and retarded electromagnetic potentials. The third time derivative in this theory is almost unique in physics. The idea that the MCM might use such a derivative was first considered in the context of the Abraham–Lorentz law with no regard for α_{MCM} or gravitation. This use case for the third derivative predated the similar requirement derived from the analytical form of α_{MCM} [30].

The Ontological Basis A labeling basis

$$\hat{\pi} \equiv \hat{e}_{\mathcal{H}} \quad , \quad \hat{2} \equiv \hat{e}_{\mathcal{A}} \quad , \quad \hat{\Phi} \equiv \hat{e}_{\Omega} \quad , \quad \text{and} \qquad \hat{i} \equiv \hat{e}_{\varnothing} \quad , \tag{1.10.52}$$

was introduced to associate the usual $|\psi\rangle$ analytical formalism with the $\{t_\star, t_+, t_-, \text{bounce}\}$ language:

$$\begin{aligned} \big|\psi;\hat{\pi}\big\rangle &= \psi(x^i, x^0) \\ \big|\psi;\hat{2}\big\rangle &= \psi(x^i_-, x^0_-) \\ \big|\psi;\hat{\Phi}\big\rangle &= \psi(x^i_+, x^0_+) \\ \big|\psi;\hat{i}\big\rangle &= \psi(x^i_\varnothing, x^0_\varnothing) \quad . \end{aligned} \tag{1.10.53}$$

This form of the $\hat{e}_\mu$ basis was called *the ontological basis* in reference to an intention to explain certain natural quantities with unique number-theoretical assignments. Future work may explore an alternative convention for $\hat{\tau}$ with $\tau = 2\pi$.

The Fine Structure Constant It was determined that the numbers in the chosen basis can be used to construct the dimensionless quantum electrodynamic coupling constant to within about 0.4% [30]:

$$\alpha^{-1}_{\text{MCM}} = 2\pi + \big(\Phi\pi\big)^3 \quad . \tag{1.10.54}$$

While some will cite the notion that α_{QED} is known to an accuracy far exceeding the

0.4% discrepancy with α_{MCM}, we have demonstrated in Section 1.9.4 that such precision does not rule out α_{MCM}. The cubed term in α_{MCM} was noted for its consistency with a hypothetical ∂_t^3 operator invoked through the context for retrocausality.

Mechanical Precession of Spinning Discs An independent but contemporaneous inquiry into the physics of spinning discs quickly led to Laithwaite's suggestion that the time derivative of acceleration might be used to explain the anomalous anti-gravity effects observed in spinning discs [97, 98]. After already seeing this rare derivative twice, calculations were made to determine what might result if $\hat{M}^3 \propto \partial_t^3$.

Einstein's Equation Following the novel result regarding α_{MCM}, the *triply supported* ∂^3 form for $\hat{M}^3$ yielded a second novel numerical result: the dimensionless constant 8π well known from Einstein's equation

$$8\pi T_{\mu\nu} = G_{\mu\nu} + g_{\mu\nu}\Lambda \quad . \tag{1.10.55}$$

The third derivative had already been under consideration, as had the number-theoretical basis $\{\hat{\pi}, \hat{\imath}, \hat{\Phi}\}$ by which this equation was derived in [3]. There was no intention beforehand to show anything related to GR. As this was the second famous dimensionless constant derived with ∂^3 and the ontological numbers, and because it appeared while examining unrelated theoretical processes, more significance was assigned to the result than would have been assigned to a similar result appearing in isolation. The appearance of one such number is easily written off as meaningless coincidence. Two physically significant, dimensionless numbers are written off less easily.

The Ontological Resolution of the Identity Following the initial derivation of Einstein's equation, a third famous dimensionless coupling constant appeared with the addition of $\hat{e}_{\varnothing} = \hat{2}$:[1]

$$\hat{1} = \frac{1}{4\pi}\hat{\pi} - \frac{\varphi}{4}\hat{\Phi} + \frac{1}{8}\hat{2} - \frac{i}{4}\hat{\imath} \quad . \tag{1.10.56}$$

In certain natural units, 4π is the dimensionless constant attached to the Poisson equations for Newtonian gravity and classical electromagnetism:

$$\rho = \frac{1}{4\pi}\nabla^2\phi \quad , \quad \text{and} \qquad J^\mu = \frac{1}{4\pi}\eta^{\mu\nu}\partial_\nu\partial_\lambda A^\lambda \quad . \tag{1.10.57}$$

It is hoped that the ontological resolution of the identity will have vast applications toward unifying disparate forces of physics. Here, ontology refers to the theory that

[1]Following the introduction of $\hat{2}$, the respective assignments of $\hat{\imath}$ and $\hat{2}$ to the $\mathcal{A}$- and $\varnothing$-branes were swapped.

the number-theoretical properties of $\{\hat{\pi}, \hat{i}, \hat{\Phi}, \hat{2}\}$ should pertain to fundamental quantities observed in nature.

1.11 Elliptic Curves and the Total Evolution Operator

In this section, we will examine a few properties and iterations of the expected total evolution operator [3, 30]

$$\hat{\Upsilon} = \hat{\mathcal{U}} + \hat{M}^3 \quad . \tag{1.11.1}$$

We will emphasize an equation for joint chronological and chirological evolutions such that

$$\left|\psi, t_0; \hat{\pi}^0\right\rangle \quad \longrightarrow \quad \left|\psi, t_1; \hat{\pi}^1\right\rangle \quad . \tag{1.11.2}$$

Then we will show that the cumulative body of MCM material suggests elliptic curves as the solutions to the evolution equation for $\hat{M}^3$ and $\hat{\Upsilon}$. In the absence of a definite equation for $\hat{\Upsilon}$ or $\hat{M}^3$, however, that which can be said about them is limited. Much of this section will discuss what does not work before we suggest in Section 1.11.5 that the missing equation is an elliptic curve, or like an elliptic curve.

1.11.1 The Original Proposal for $\hat{\Upsilon}$

The context for $\hat{M}^3$ in the previous sections has been motivated by the three steps of $\mathcal{H} \to \Omega \to \mathcal{A} \to \mathcal{H}$ inherent to the unit cell. The original motivation for $\hat{M}^3$ (Appendix A) was that some operator should return the cubed term in α^{-1}_{MCM}. $\hat{\Upsilon}$ was formulated in [3, 30] to return α^{-1}_{MCM} as the sum of $\hat{M}^3$ with another operator appearing in its first power. The $\Phi\pi$ term was expected to be associated with some new mechanism since Φ does not usually appear in QM. The only reasonable choice for the linear derivative returning 2π was ∂_x. This is the momentum operator divided by a constant but that operator lacked the complexity required for a new role in physics. Namely, $\hat{\Upsilon}$ was envisioned as an evolution operator returning α^{-1} as a characteristic of some ontological evolution in the way that the unitary evolution operator $\hat{\mathcal{U}}$ returns e^{iEt} when $\hat{H}$ does not depend on t. The value for α should be universal because the evolution generated by $\hat{\Upsilon}$ would reflect some universal structure underlying time evolution. The 2D spacetime box first proposed for that structure has been deprecated[1] but the modern thinking remains the same: α should characterize the unit cell.

Rather than choosing the ∂_x derivative directly, we selected $\hat{\mathcal{U}}$. The idea was that

[1] Schrödinger's original derivation of his equation from the stationary action principle [80] has been deprecated even while the underlying equation remains the same. Presently, the invariant equation is $\alpha^{-1}_{\text{MCM}} = 2\pi + (\Phi\pi)^3$.

$\hat{\mathcal{U}}$ is proportional to ∂_x as

$$\hat{\mathcal{U}}(t,t_0)=\exp\left\{\frac{-i\hat{H}\big(t-t_0\big)}{\hbar}\right\}\quad,\quad\text{with}\qquad \hat{H}=\frac{-\hbar^2}{2m}\,\partial_x^2+\hat{V}\quad, \tag{1.11.3}$$

and $\hat{M}^3$ should be a complementary operator proportional to the time derivative. This was written as

$$\hat{\mathcal{U}}\propto\partial_x\quad,\quad\text{and}\qquad \hat{M}\propto\partial_t\quad. \tag{1.11.4}$$

Whatever formalism might have selected ∂_x from $\hat{H}$ with a square root, as well as the likelihood that $\hat{\Upsilon}$ would have returned $e^{\alpha_{\rm MCM}^{-1}}$, was left to omitted details via the := relationship. The relations between in (1.11.4) were originally stated with the := symbol meaning "is defined according to" rather than the present $\propto$ symbol meaning "proportional to."

The case for using $\hat{\mathcal{U}}$ was mainly to force an association of the $\hat{\Upsilon}$ operator with time evolution, and to frame $\hat{M}^3$ as a new kind of time evolution operator. However, the reference to $\hat{\mathcal{U}}$ in early work rather than simply writing $\hat{\Upsilon}=\partial_x+\hat{M}^3$ may have clouded the intended meaning. Heavy reliance on the := formalism to omit less important ancillary mathematical details may have also hindered what was intended to be a rapid communication [3, 30].

1.11.2 $\hat{\Upsilon}$ Redefined with ∂_0 and ∂_4

Work subsequent to [3, 30] introduced the χ^4 variables which alter the possibilities for $\hat{\Upsilon}$. Namely, if $\hat{\mathcal{U}}$ is the chronological time evolution operator then $\hat{M}^3$ should complement it as the chirological time evolution operator:

$$\begin{matrix}\hat{\mathcal{U}}\propto\partial_x \\ \hat{M}^3\propto\partial_t\end{matrix}\qquad\xrightarrow{\text{New }\chi^4\text{ variables}}\qquad\begin{matrix}\hat{\mathcal{U}}:\partial_0 \\ \hat{M}^3:\partial_4^3\quad.\end{matrix} \tag{1.11.5}$$

It was pointed out in Section 1.9.3 that ∂_x can return 2π in a problem of one spatial dimension but the more realistic ∇ operator in 3D will return a sum of three terms incompatible with $\alpha_{\rm MCM}$. The introduction of ∂_4 allows us to avoid this problem. The spatial derivative does not appear in the new relationships $\hat{\mathcal{U}}:\partial_0$ and $\hat{M}^3:\partial_4^3$. Furthermore, removing ∂_i from consideration motivates the universality of the returned value α because we have eliminated non-universal contributions from arbitrary $V(x)$ potential energy landscapes. The constant width of the unit cell in the abstract coordinates for any chronological time step between measurements sketches a good reason for why the total evolution operator should return a constant value in arbitrary systems.

The meaning on the right in (1.11.5) is that ∂_0 acts on $\hat{\mathcal{U}}$ in Schrödinger's equation as

$$i\hbar\partial_0\hat{\mathcal{U}} = \hat{H}\hat{\mathcal{U}} \quad , \tag{1.11.6}$$

and $\hat{M}^3$ is meant to complement that as, for example,

$$\hat{\Upsilon} = i\hbar\partial_0\hat{\mathcal{U}} + \partial_4^3\hat{M}^3 \quad . \tag{1.11.7}$$

A first guess for an equation for $\hat{\Upsilon}$ would be

$$\partial_4^3\hat{M}^3 + i\hbar\partial_0\hat{\mathcal{U}} = \hat{H}_{\text{MCM}}\big(\hat{\mathcal{U}} + \hat{M}^3\big) \quad . \tag{1.11.8}$$

In presenting this guess, we demonstrate what was meant when it was written in [3] that that $\hat{M}^3$ should "complement" $\hat{\mathcal{U}}$. The reader is also reminded that Schrödinger's equation comes "out of the mind of Schrödinger," as Feynman puts it [99], and nowhere else. It cannot be derived from first principles and it is not expected that first principles analysis might conclude with a new total evolution equation for $\hat{\Upsilon}$. Rather, (1.11.8) is an example of a new equation for $\hat{M}^3$ which should reduce to Schrödinger's equation in the limit vanishing χ^4 and vanishing derivatives with respect to χ^4. The vanishing derivative removes $\hat{M}^3$ on the left and vanishing χ^4 should remove it on the right. Without those terms, (1.11.8) is the Schrödinger equation in $\mathcal{H}$.

1.11.3 The Schrödinger Equation for $\hat{\Upsilon}$

Considering $\hat{\Upsilon}$ as it was before χ^4, Schrödinger's equation for $\hat{\mathcal{U}} + \hat{M}^3$ is

$$i\hbar\partial_0\big(\hat{\mathcal{U}} + \hat{M}^3\big) = \hat{H}\big(\hat{\mathcal{U}} + \hat{M}^3\big) \quad . \tag{1.11.9}$$

If $\hat{H}$ is a pre-MCM Hamiltonian, then (1.11.9) is separable as

$$i\hbar\partial_0\hat{\mathcal{U}} = \hat{H}\hat{\mathcal{U}} \quad , \qquad \text{and} \qquad i\hbar\partial_0\,\hat{M}^3 = \hat{H}\hat{M}^3 \quad . \tag{1.11.10}$$

The condition that $\hat{H}$ is "pre-MCM" means that $i\hbar\partial_0\hat{\mathcal{U}}=\hat{H}\hat{\mathcal{U}}$ is valid. If $\hat{M}^3$ depends on x^0, then it is equal to $\hat{\mathcal{U}}$. If it does not depend on x^0, then $\partial_0\hat{M}^3=0$ and it follows that $\hat{M}^3=0$ or $\hat{H}=0$. These results are not useful. Under the naive operation of $\hat{M}^3$ as a translation operator, we have

$$\begin{aligned}\hat{\Upsilon}\big|\psi,t_0;\hat{\pi}^0\big\rangle &= \hat{\mathcal{U}}\big|\psi,t_0;\hat{\pi}^0\big\rangle + \hat{M}^3\big|\psi,t_0;\hat{\pi}^0\big\rangle \\ &= \big|\psi,t_1,\hat{\pi}^0\big\rangle + 2\pi\Phi\big|\psi,t_0;\hat{\pi}^1\big\rangle \quad .\end{aligned} \tag{1.11.11}$$

The orthogonality of MCM plane waves is such that wavefunctions in $\hat{\pi}^k$ cannot interfere with those in $\hat{\pi}^j$ if $k \neq j$. They are linearly independent. If such states did interfere, then (1.11.11) could in principle yield one coherent probability amplitude for a state at time t_1 on level $\hat{\pi}^1$. However, the orthogonality of wavefunctions on different levels of aleph is required for other applications and we should not suppose that they might not be orthogonal. Using $\hat{M}^3$ as a translation operator, a coherent amplitude with the correct t and $\hat{\pi}^k$ specifiers is generated by

$$\hat{\mathcal{U}}(t,0)\hat{M}^3\big|\psi,0;\hat{\pi}^0\big\rangle = 2\pi\Phi\big|\psi,t;\hat{\pi}^1\big\rangle \quad . \tag{1.11.12}$$

Schrödinger's equation for $\hat{\mathcal{U}}\hat{M}^3$ is

$$\begin{aligned} i\hbar\partial_0\big|\psi,t;\hat{\pi}^1\big\rangle &= \hat{H}\big|\psi,t;\hat{\pi}^1\big\rangle \\ \frac{i\hbar}{2\pi\Phi}\,\partial_0\hat{\mathcal{U}}\hat{M}^3\big|\psi,0;\hat{\pi}^0\big\rangle &= \hat{H}\,\hat{\mathcal{U}}\hat{M}^3\big|\psi,0;\hat{\pi}^0\big\rangle \\ \frac{i\hbar}{2\pi\Phi}\,\partial_0\big(e^{-i\hat{H}t/\hbar}\hat{M}^3\big) &= \hat{H}\,e^{-i\hat{H}t/\hbar}\hat{M}^3 \quad . \end{aligned} \tag{1.11.13}$$

If $\hat{M}^3$ does not depend on t, then

$$\hat{M}^3 = \exp\left\{\frac{-i\hat{H}\big(2\pi\Phi-1\big)t}{\hbar}\right\} \quad . \tag{1.11.14}$$

This $\hat{M}^3$ combines with $\hat{\mathcal{U}}$ as

$$\hat{\mathcal{U}}\hat{M}^3 = \exp\left\{\frac{-2\pi i\Phi\hat{H}t}{\hbar}\right\} \quad . \tag{1.11.15}$$

We have added no physics with this equation. If not for the $2\pi\Phi$ scale factor, we would have found $\hat{M}^3 = \mathbb{1}$. $\hat{M}^3$ is still executing some form of equal-time parallel transport, albeit complemented with the $\hat{\mathcal{U}}$ operator.

Since the $\hat{x}$ and $\hat{p}$ operators don't commute, (1.11.15) begs that we ask about the commutation relations of $\hat{\mathcal{U}}$ and $\hat{M}^3$. If $[\hat{\mathcal{U}},\hat{M}^3]=0$ (assumed in (1.11.13)), the chronological time step can be implemented anywhere during the transit of the unit cell. There would be no difference between landing on Ω or $\mathcal{A}$ at t_0 or t_1. This is not the desired behavior because it mitigates the dynamical uniqueness which the intermediate steps at Ω and $\mathcal{A}$ were introduced to generate. Clearly, more physics is required. To the extent that the product $\hat{\mathcal{U}}\hat{M}^3$ seems better suited than $\hat{\mathcal{U}}+\hat{M}^3$ towards total evolutions in the form $|\psi,t_0,\hat{\pi}^0\rangle \to |\psi,t_1,\hat{\pi}^1\rangle$, we might consider $\hat{\Upsilon}$ such that

$$e^{-i\hat{\Upsilon}} = e^{-i\hat{H}t/\hbar}e^{-i\hat{M}^3} \quad . \tag{1.11.16}$$

In this way of writing $\hat{\Upsilon}$, $\hat{M}^3$ complements $\hat{\mathcal{U}}$'s generator $\hat{H}$ rather than $\hat{\mathcal{U}}$ itself. Here again, $\hat{M}^3$ is only a new energy term of the sort discussed in Section 1.7.4.

1.11.4 Total Evolution by $\hat{\Upsilon}$

Toward an evolution equation, the $i\partial_0$ part of

$$\hat{\alpha} = i\big(\partial_0 - \partial_4^3\big) \quad ,^1 \tag{1.11.17}$$

is already in Schrödinger's equation so

$$\hat{\alpha}\big|\psi\big\rangle = i\big(\partial_0 - \partial_4^3\big)\big|\psi_\alpha\big\rangle = \hat{H}_{\text{MCM}}\big|\psi_\alpha\big\rangle \quad , \tag{1.11.18}$$

is a good lead toward an equation for $\hat{M}^3$. It contains a third derivative and, given an appropriate $\hat{H}_{\text{MCM}}$, it almost reduces to Schrödinger's equation in the limit of vanishing χ^4 and ∂_4 derivatives. The only disagreement in that limit is the missing factor of $\hbar$. On that count, the units of (1.11.17) were not right to begin with. ∂_0 has units of inverse seconds but ∂_4^3 probably does have those units. Indeed, the equation

$$i\big(\partial_0 - \partial_4^3\big)\big|\psi_\alpha\big\rangle = \Big[2\pi + \big(\Phi\pi\big)^3\Big]\,\big|\Psi_\alpha\big\rangle \quad , \tag{1.11.19}$$

does not return a manifestly dimensionless α^{-1}_{MCM}. Likewise , the operator on the left side of (1.11.18) is supposed to be dimensionless but the returned value on the right is an energy. Given these problems with physical units, and given that the values 2π and $\Phi\pi$ must be fixed in Ψ_α when the wavenumber and frequency are usually allowed to vary in physical states, we might write an equation totally in the abstract coordinates:

$$i\left[\frac{\partial}{\partial\chi_0} - \frac{\partial^3}{\partial\chi_4^3}\right]\big|\Psi'_\alpha\big\rangle = -\frac{1}{2}\left[\frac{\partial^2}{\partial\chi_1^2} + \frac{\partial^2}{\partial\chi_2^2} + \frac{\partial^2}{\partial\chi_3^2}\right]\big|\Psi'_\alpha\big\rangle \quad . \tag{1.11.20}$$

This equation in which we have lowered the tensor indices for convenience sets $\hat{H}_{\text{MCM}}$ as the chirological free particle Hamiltonian with $\hbar = m = 1$, and we have replaced Ψ_α with

$$\Psi'_\alpha(\chi^0,\chi^i,\chi^4) = \exp\big\{ -i\big(2\pi\chi^0 + \Phi\pi\chi^4 - k_i\chi^i\big)\big\} \quad . \tag{1.11.21}$$

We have not previously referred to the χ^0 and χ^i abstract coordinates. However,

[1] This operator fist appeared as (1.9.6).

the question about the universality of the returned value for α is well wrapped up when we suppose a new equation which only involves derivatives with respect to the abstract coordinates.

1.11.5 Elliptic Curves

Elliptic curves are third order functions of two variables. Those in Figure 17 are of the form

$$y^2 = x^3 + ax + b \quad . \tag{1.11.22}$$

The two lower figures show the behavior of the Riemann ζ function near the $z=\infty$ north pole of the Riemann sphere [46,48]. Given a statement of $\hat{\Upsilon}$ in which we obtain α_{MCM} by modifying the time part of Schrödinger's equation as

$$-i\big(\partial_4^3 - \partial_0\big)\psi = \hat{H}\psi \quad , \tag{1.11.23}$$

the remarkable likeness in Figure 17 is *quite remarkable*. The affine parameter along a smooth curve through $\Sigma^{\pm}$ connecting two $\mathcal{H}$-branes must increase monotonically between $\mathcal{H}_0$ and $\mathcal{H}_1$, as must the chronological time. Therefore, we are invited to parameterize χ^4 in terms of x^0. Choosing $\chi^4 = \tau x^0$ allows us to rewrite (1.11.23) as

$$-i\big(\tau^3\partial_0^3 - \partial_0\big)\psi = \left(\frac{-\hbar^2}{2m}\partial_x^2 + \hat{V}\right)\psi \quad . \tag{1.11.24}$$

By introducing new variables

$$\begin{aligned} x &= i\tau\partial_0 \\ a &= \tau^{-1} \\ y &= \frac{i\hbar}{\sqrt{2m}}\partial_x \\ b &= -\hat{V} \quad , \end{aligned} \tag{1.11.25}$$

we obtain

$$\big(x^3 + ax + b\big)\psi = \big(y^2\big)\psi \quad . \tag{1.11.26}$$

This equation must be compared to (1.11.22). The ansatz equation (1.11.23) whose left side contains the $\hat{\alpha} \sim \hat{\Upsilon}$ operator used to return α_{MCM}^{-1} is almost an identical elliptic curve. It is a differential equation whose characteristic curves or auxiliary equations are likely to be elliptic curves in the form of (1.11.22). The likeness of the two equations mirrors that between the classical dispersion relation and Schrödinger's

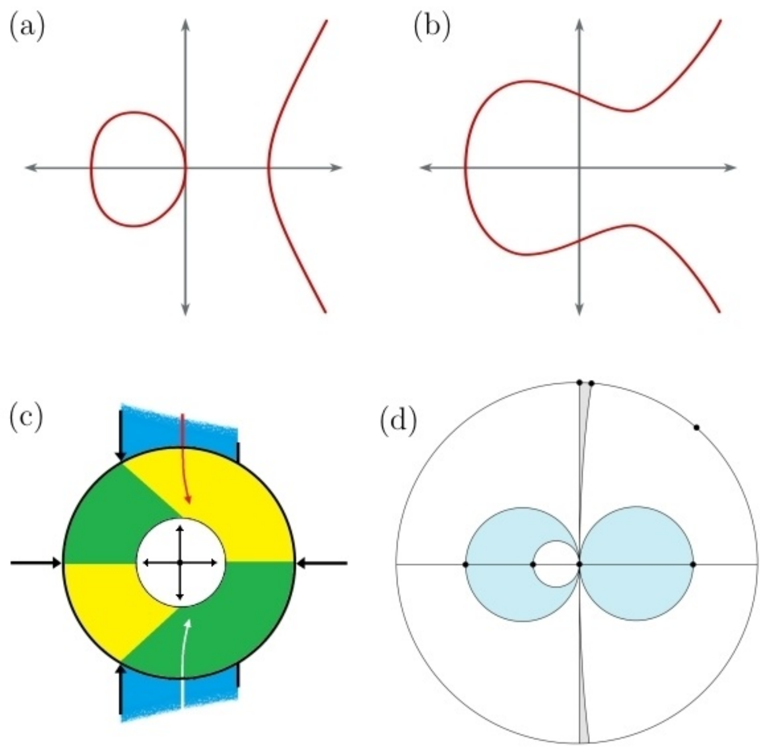

Figure 17: Above are two elliptic curves. Below are two figures showing the behavior of the Riemann ζ function around the north pole of the Riemann sphere. (a) The curve $y^2 = x^3 - x$. (b) The curve $y^2 = x^3 - x + 1$. (c) This figure is taken from [48] wherein the negation of the Riemann hypothesis was laid out in principle. (d) This figure is taken from [46], one of a few papers in which independent, formal negations of the Riemann hypothesis are given. The left-right asymmetry of (a) and (b) is qualitatively very similar to that in (c) and (d).

equation,

$$\omega - \frac{k^2}{2m} = 0 \quad , \qquad \text{and} \qquad \left(i\hbar\partial_t + \frac{\hbar^2}{2m}\nabla^2 \right)\psi = 0 \quad , \tag{1.11.27}$$

under the change of variables

$$\omega \to i\hbar\partial_t \quad , \qquad \text{and} \qquad k \to i\hbar\nabla \quad . \tag{1.11.28}$$

Fundamental physical equations are usually simple representations of classes of equations so it is likely that the equation for $\hat{M}^3$ will be a well known equation in elliptic curve analysis. Complicating factors on the path to finding the *exact* equation include the singularity $\widehat{\infty}$ appearing in $\varnothing$. The elliptic curves in Figure 17 are depicted near the origin but the MCM curves are depicted near the opposite pole of the Riemann sphere at $z=\infty\notin\mathbb{C}$ (or $z=\widehat{\infty}\notin\mathbb{C}$.) This is likely to induce new complexity into the problem which may exceed the usual study of elliptic curves. Namely, the parameterization which allows us to replace ∂_4 with $\tau^{-1}\partial_0$ must go through a singularity at $\varnothing$. However, if we cast the equation with ∂^4 and ∂_4^3, meaning that we parameterize x^0 in terms of χ^4 rather than vice versa, we might avoid the physical singularity by remaining in the abstract coordinates.[1] In that case, the reversal of time arrows and the multiple dislocated origins for the piecewise $\chi^4_\pm$ and $\chi^4_\varnothing$ elements of what is only *called* χ^4 may complicate an assumption that χ^4 and x^0 monotonically increase in tandem between two $\mathcal{H}$-branes. This assumption is required for the affine parameterization of one by the other with a linear expression.

To emphasize that which is most intriguing in Figure 17, and to work toward dispelling any suggestion that the similar quality in the figure is meaningless in the way that certain qualia pertaining to $\hat{M}^3$ are said to be meaningless, consider Wiles' statement regarding the Birch and Swinnerton-Dyer conjecture [100].

> "Mathematicians have always been fascinated by the problem of describing all solutions in whole numbers x, y, z to algebraic equations like
>
> $$x^2+y^2=z^2 \quad . \tag{1.11.29}$$
>
> Euclid gave the complete solution for that equation, but for more complicated equations this becomes extremely difficult. Indeed, in 1970 Yu. V. Matiyasevich showed that Hilbert's tenth problem is unsolvable, i.e., there is no general method for determining when such equations have a solution in whole numbers. But in special cases one can hope to say something. When the solutions are the points of an abelian variety, the Birch and Swinnerton-Dyer conjecture asserts that the size of the group of rational points is related to the behavior of an associated zeta function $\zeta(s)$ near the point $s=1$. In particular this amazing conjecture asserts that if $\zeta(1)$ is equal to 0, then there are an infinite number of rational points (solutions), and conversely, if $\zeta(1)$ is not equal to 0, then there is only a finite number of such points."

Considering that that Birch and Swinnerton-Dyer conjecture regards an object $L(C,z)$ where C is an elliptic curve, it is known that there exists at least one famous

[1] Recall that $\chi^4_\pm$ is taken as the Ricci scalar defining the dS and AdS physical $g^\pm_{\mu\nu}$ metrics when $A^\mu_\pm=0$. Thus, $\chi^4_\pm\to\pm\widehat{\infty}$ at $\varnothing$ does not necessarily require an abstract singularity at $\varnothing$. In the abstract coordinates, the location of $\varnothing$ at a non-arithmatic number is a sufficient topological obstruction between $\Sigma^\pm$.

problem of interest relating elliptic curves to ζ functions. Therefore, a condition of total and/or profound irrelevance that detractors might cite for the correspondence in Figure 17 is not the true condition. We have sufficient reason to suppose a connection between the Riemann ζ function and elliptic curves, and a further connection to $\hat{M}^3$. As to what the true condition of Figure 17 might be, the formal statement of the Birch and Swinnerton-Dyer conjecture [100] exceeds this writer's training.

Regarding the difficulty of Birch and Swinnerton-Dyer and its relevance of elliptic curves, Johnson writes the following [101].

> "There is no doubt that elliptic curves are amongst the most closely and widely studied objects in mathematics today. ***The arithmetic complexity of these particular curves is absolutely astonishing*** [*emphasis added*], so it isn't surprising the Birch and Swinnerton-Dyer conjecture has been honored with a place amongst the Clay Mathematics Institute's famous Millennium Prize Problems. Although some great unsolved problems carry the benefit of simplicity in statement, this conjecture is not one of them. There even seems to be an aura of 'hardness' over the problem that keeps many from discovering the true beauty of the conjecture. [*sic*] The Birch and Swinnerton-Dyer conjecture today remains, of course, unsolved and most mathematicians agree that new ideas will need to be developed to tackle the great problem. A proof will take a great deal of work and mathematical power."

The present problem regarding the elliptic curve application for $\hat{M}^3$'s equation requires a survey of some large volume of number theory. The work might far exceed the ordinary scope of a PhD problem.

Part II: Problems in Physics

The thesis problems in Part II are presented with less detail than the problem in Part I. These problems are mostly applications for the MCM and/or fractional distance analysis toward open problems in physics.

2 Period Doubling

This problem in mathematical physics is as described in the following excerpt from [96]. It concerns period doubling behavior in equations such as (2.1).